

Can Virtual MIMO Mimic a Multi-Antenna System?

Diversity- Multiplexing Tradeoff in Cooperative Wireless Systems

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Introduction- MIMO

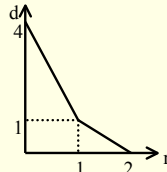
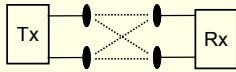
- Multiple antennas improve the performance of wireless systems significantly.
- Traditionally multiple antennas have been mainly used for increasing **diversity gains**. The diversity gain, d , shows how fast the **probability of error**, P_e , decays with increasing SNR.

$$d = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}} \text{ or } P_e(\text{SNR}) \approx \frac{1}{\text{SNR}^d} \text{ at high SNR}$$

- But multiple antennas also introduce higher spatial multiplexing gains. The multiplexing gain, r , shows how fast the **actual rate**, R , increases with increasing SNR.

$$r = \lim_{\text{SNR} \rightarrow \infty} \frac{R(\text{SNR})}{\log \text{SNR}} \text{ i.e. } R(\text{SNR}) \text{ scales like } r \log \text{SNR} \text{ at high SNR}$$

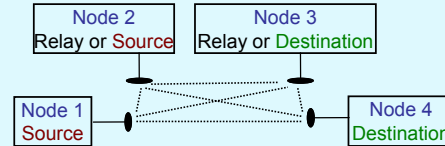
- There is a fundamental tradeoff between diversity and multiplexing gains. e.g. For a 2X2 system:



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Introduction- Cooperative Systems

- Cooperation/relaying is considered as a viable substitute for real MIMO systems to get diversity and multiplexing gains.
- The system below has the same number of antennas as a 2X2 MIMO system, **but how close can it be to a real MIMO?**



- The answer depends on the location of the nodes.
- We will investigate this problem in terms of diversity-multiplexing tradeoff.

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Clustering

Rayleigh Zone



- If in Rayleigh Zone: nodes not too close → no strong line of sight component.

AWGN Zone



- If in AWGN Zone: nodes are close → strong line of sight component: Strongly Ricean distribution is approximated by AWGN.

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Upper Bounds

- We find the upper bounds using min-cut, max-flow results.
- Best case: The nodes are full-duplex.

Relay case: A single source-destination pair, 2 relays



- Cutset 1: $R_1 \leq I(X_1; Y_2 Y_3 Y_4 | X_2 X_3)$
- Cutset 2: $R_2 \leq I(X_2; Y_3 Y_4 | X_1 X_3)$
- Cutset 3: $R_3 \leq I(X_1 X_2 X_3; Y_4)$

- If in Rayleigh Zone: Source to Relay 1 and Relay 2 to destination channels are bottlenecks.

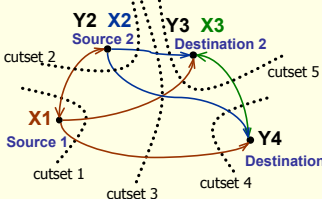
- If in AWGN Zone: Source to Relay 1 and Relay 2 to destination channels are Gaussian with **limited capacity**. This is why the **multiplexing gain is limited to 1**. In a real MIMO, these channels have infinite capacity.

X_i : transmitted signal
 Y_i : received signal

Cooperative case: Two sources-two destinations

Both destinations decode both sources

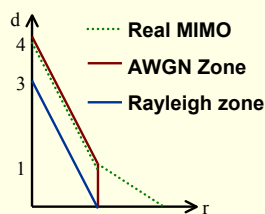
This case represents information transfer from a group of antennas to another group and is analogous to MIMO.



- Cutset 1: $R_1 \leq I(X_1; Y_2 Y_3 Y_4 | X_2 X_3 X_4)$
- Cutset 2: $R_2 \leq I(X_2; Y_1 Y_3 Y_4 | X_1 X_3 X_4)$
- Cutset 3: $R_1 + R_2 \leq I(X_1 X_2; Y_3 Y_4 | X_3 X_4)$
- Cutset 4: $R_1 + R_2 \leq I(X_1 X_2 X_3; Y_4)$
- Cutset 5: $R_1 + R_2 \leq I(X_1 X_2 X_4; Y_3)$

- Sum multiplexing gain is again limited to 1 as the finite capacity link between the destinations become limiting.

- Even if the cooperative/relay system is clustered, it can never provide all the gains of a real MIMO. The spatial multiplexing gain is limited to 1!
- This holds true if the destinations are interested in corresponding sources only, but not both.
- This is an upper bound, how do we achieve this tradeoff?



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Achievability

Relay case:

• **AWGN Zone**

- Relay 1 does decode-and-forward and Relay 2 does compress-and-forward with channel side information at the destination. [This is the *mixed strategy of Kramer, Gastpar, Gupta 2005.*]

- This way the source and relay 1 act like a transmit antenna array and relay 2 and the destination act like a receive antenna array. Hence, the system can provide real MIMO diversity gains for all multiplexing gains ≤ 1 .

• **Rayleigh Zone**

- Both relays do decode-and-forward. This simpler scheme is enough to obtain the diversity-multiplexing gains.

Block 1	Block 2	Block 3	Block 4
$x_1(\omega_1)$	$x_1(\omega_1, \omega_2)$	$x_1(\omega_2, \omega_3)$	$x_1(\omega_3, 1)$
$x_2(1)$	$x_2(\omega_1')$	$x_2(\omega_2')$	$x_2(\omega_3')$
$x_1(1)$	$x_1(\omega_1'')$	$x_1(\omega_2'')$	$x_1(\omega_3'')$

Cooperative case: Both destinations decode both sources.

• **AWGN Zone**

- Both sources decode each others information. [Willems, van der Meulen, Schalkwijk 1983] The destinations mutually compress-and-forward with channel state information at the destination.
- When the sources and destinations are paired and the destinations only decode their corresponding sources, this scheme still achieves the diversity multiplexing tradeoff upperbound.

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Conclusions and Future Work

- A cooperative/relay system always falls short of a real MIMO. Although a real 2X2 MIMO can support multiplexing gains as high as 2, a cooperative/relay system is limited to multiplexing gain 1.
- We are currently working on systems with half-duplex nodes.
- Future work includes extending the work to systems with multiple-antenna elements.

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