

Performance of Amplify-and-forward and Decode-and-Forward Relaying with Turbo Codes

Michael Souryal*

NIST

**National Institute of
Standards and Technology**

Technology Administration
U.S. Department of Commerce

Branimir Vojcic†

THE GEORGE

WASHINGTON

UNIVERSITY

WASHINGTON DC

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Cooperative Protocols Studied & Objectives

- Amplify-and-forward (AF)
- Fixed decode-and-forward (DF)
- Adaptive decode-and-forward (ADF)
(source retransmits if relay cannot successfully decode)
- *Hybrid AF/DF* (DF if relay can decode; otherwise, AF)

Objectives

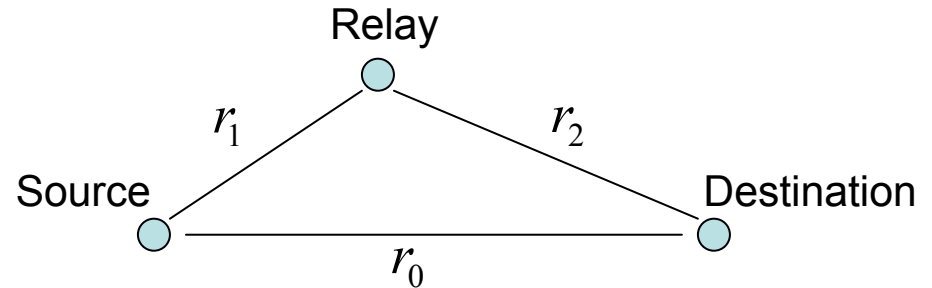
1. Analytical expressions for frame error rate (FER)
2. Performance comparison using a practical turbo code

Main Findings

1. Exact expression for distribution of SNR with AF
2. Hybrid AF/DF better than AF and close to ADF over a range of channels

System Model

- Time-division transmission
- Code repetition
- Channel:
 - Path loss with distance
 - Quasi-static fading, mutually indep. links



- Demodulator sampled output:

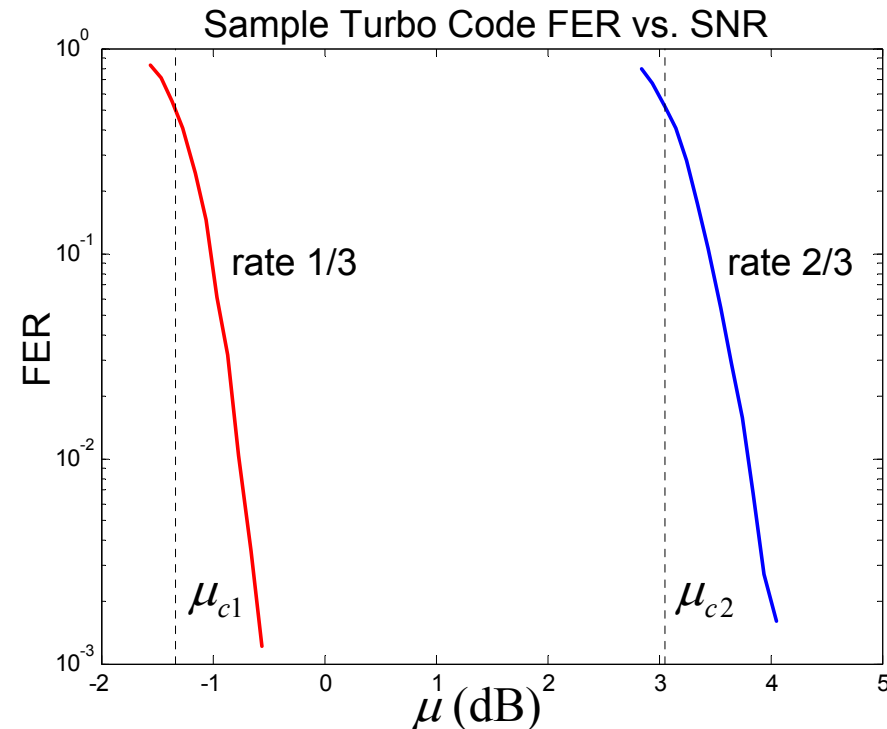
$$y_i = \alpha_i s_i + z_i$$

$E|\alpha_i|^2 = 1/r_i^n$ (complex Gaussian, $\sigma^2 = N_0/E_s$)

- Threshold-SNR model for successful reception:

$$P_s = \Pr[\mu > \mu_c]$$

received SNR / SNR threshold



Distribution of Amplify-&-Forward SNR

- SNR at destination after maximal ratio combining:

$$\mu_{\text{AF}} = \frac{1}{\sigma^2} \left(\Gamma_0 + \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2 + \sigma^2} \right) \quad ; \quad \Gamma_i = |\alpha_i|^2 \sim \text{Exp}(r_i^n)$$

(Rayleigh fading)

- Distribution function of μ_{AF} :

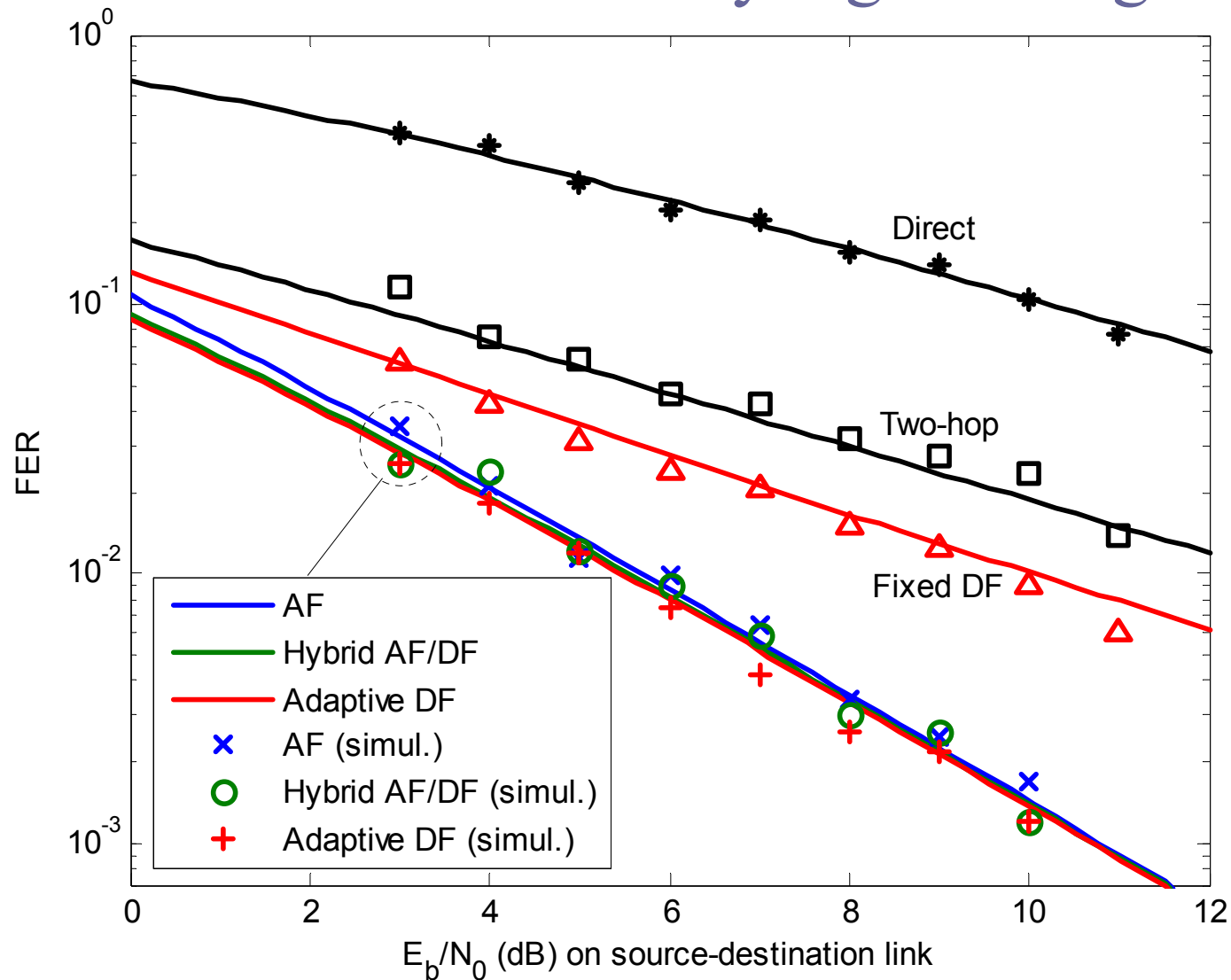
$$F_{\text{AF}}(\mu) = 1 - e^{-\sigma^2 r_0^n \mu} - r_0^n e^{-\sigma^2 r_0^n \mu} \int_0^{\sigma^2 \mu} e^{-(r_1^n + r_2^n - r_0^n)t} g(t) K_1(g(t)) dt$$

where $K_1(\cdot)$ is the first-order modified Bessel fn. of the second kind, and $g(t) = 2\sqrt{r_1^n r_2^n t(t + \sigma^2)}$

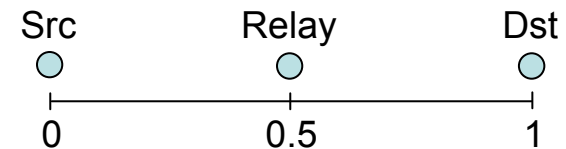
- Success probability with AF cooperation:

$$P_{s,\text{AF}} = \Pr[\mu_{\text{AF}} > \mu_{c2}] = 1 - F_{\text{AF}}(\mu_{c2})$$

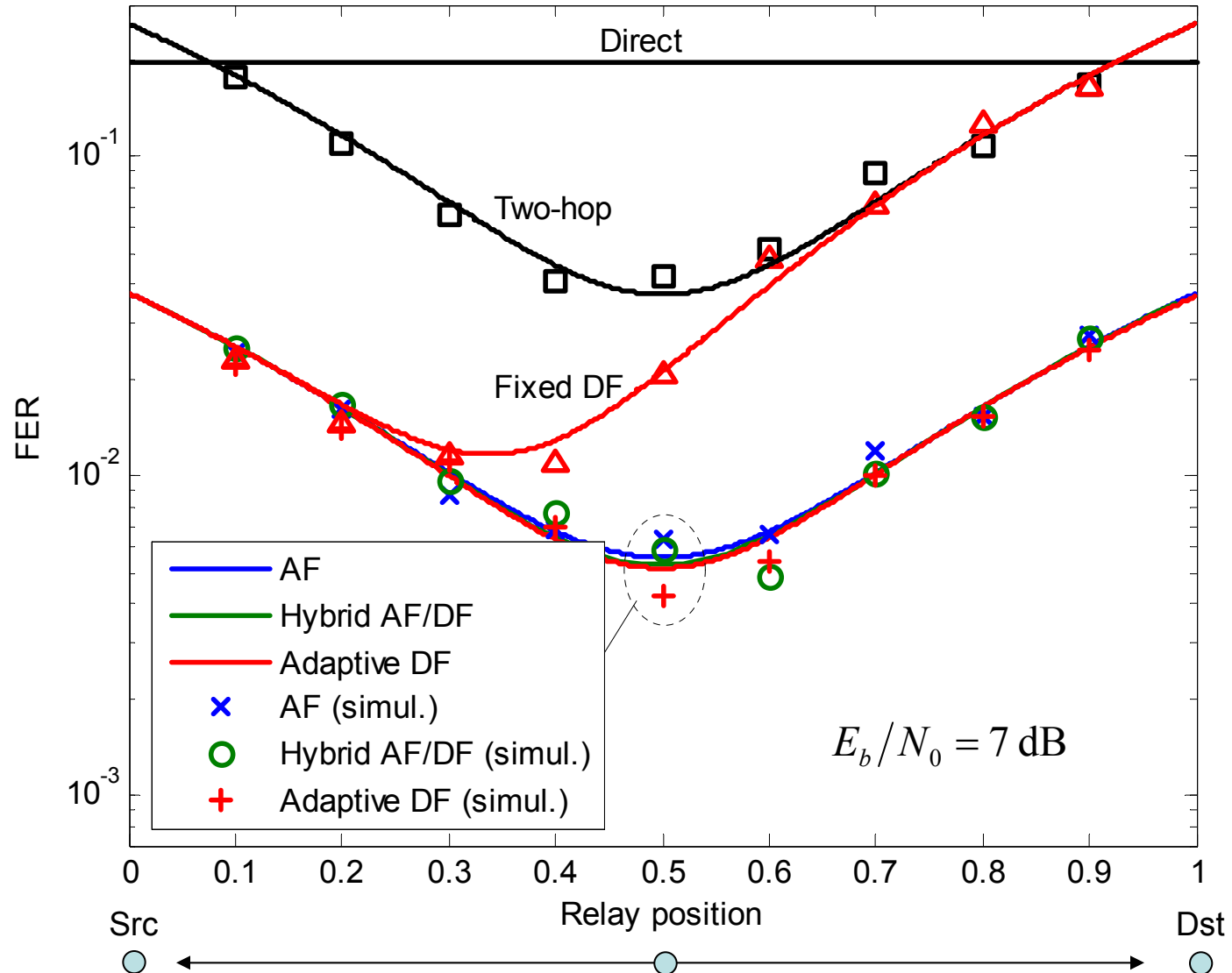
FER vs. SNR in Rayleigh Fading



Lines are analytical results. Markers are simulation results.

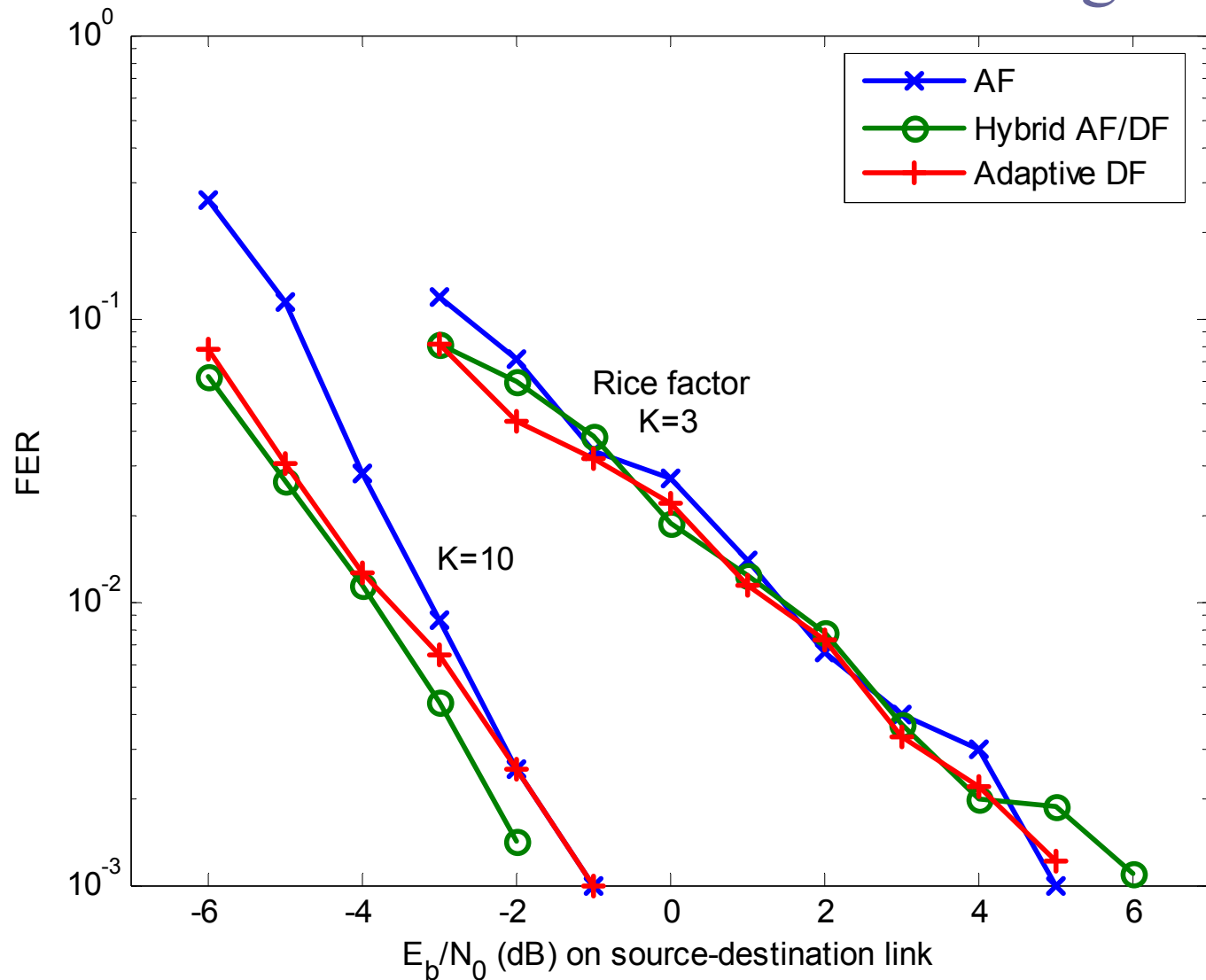


FER vs. Relay Position, Rayleigh Fading

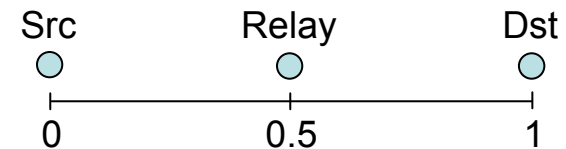


Lines are analytical results. Markers are simulation results.

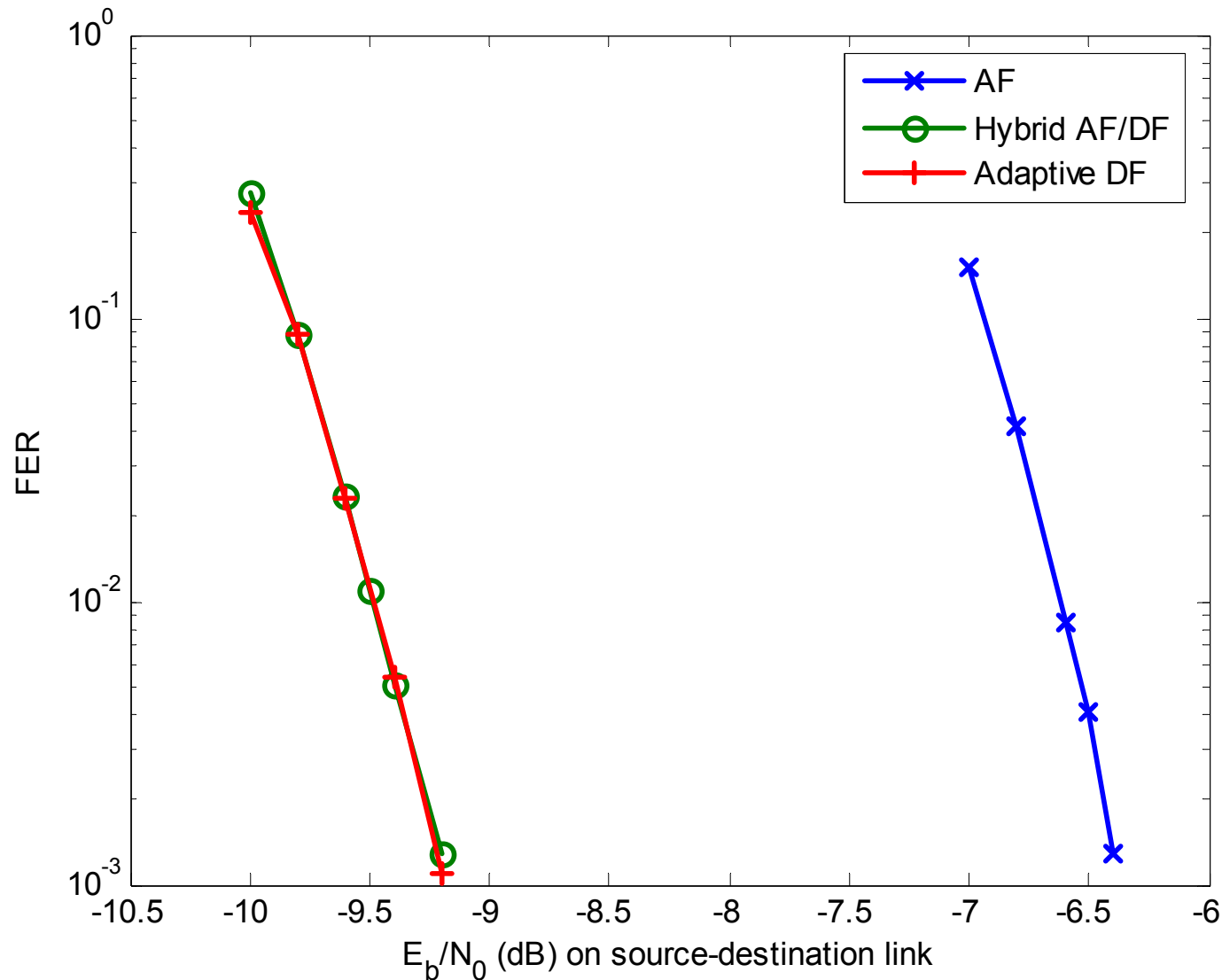
FER vs. SNR in Ricean Fading



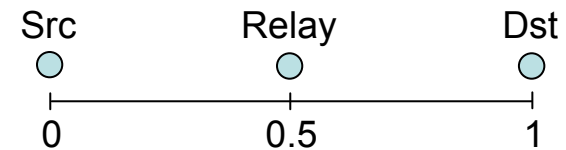
Simulation results



FER vs. SNR in AWGN



Simulation results



Summary & Conclusions

- AF, ADF, and Hybrid AF/DF all achieve diversity and perform comparably in Rayleigh fading.
- Advantages of Hybrid AF/DF:
 - Lower protocol overhead than ADF
 - Better performance than AF in specular channels
- Optimum relay position is halfway between source and destination.

Related work:

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [2] E. Zimmermann, P. Herhold, and G. Fettweis, “On the performance of cooperative diversity protocols in practical wireless systems,” in *Proc. Vehicular Technology Conf. (VTC)*, vol. 4, Fall 2003, pp. 2212–2216.
- [3] M. Yu and J. Li, “Is amplify-and-forward practically better than decode-and-forward or vice versa?” in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 3, March 2005, pp. 365–368.