Randomized Space-Time Coding for Distributed Cooperative Communication

Birsen Sirkeci Mergen and Anna Scaglione

Cornell University

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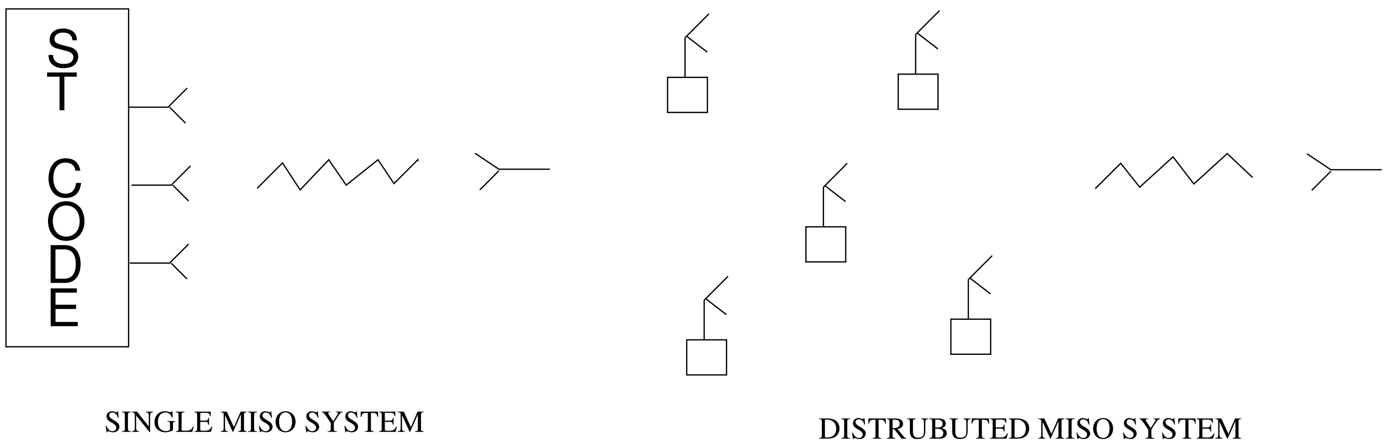
Motivation

Problem: Decentralizing the Space-time (ST) coding policy:



- Orthogonal space-time codes are proposed to achieve diversity by Laneman et al [2003] $\Rightarrow$ codes have to be assigned to each node.
- Proposed scheme: Randomized space-time codes $\Rightarrow$ each node uses a random linear combination of the columns of an underlying ST code.

Randomized ST Coding

- Let $\mathbf{s} = [s_0 s_1 \dots s_{n-1}]$ be the block of source symbols
- $\boldsymbol{\mathcal{G}}(\mathbf{s})$: $P \times L$ ($P \geq L$) underlying space-time code (L virtual antennas)
- IDEA: The randomized space-time coding is a double mapping:

$$\mathbf{s} \longrightarrow \boldsymbol{\mathcal{G}}(\mathbf{s}) \longrightarrow \boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}},$$

$\boldsymbol{\mathcal{R}} = [\mathbf{r}_1 \dots \mathbf{r}_N]$, $\mathbf{r}_i$ is the random coefficient vector for the i'th node.

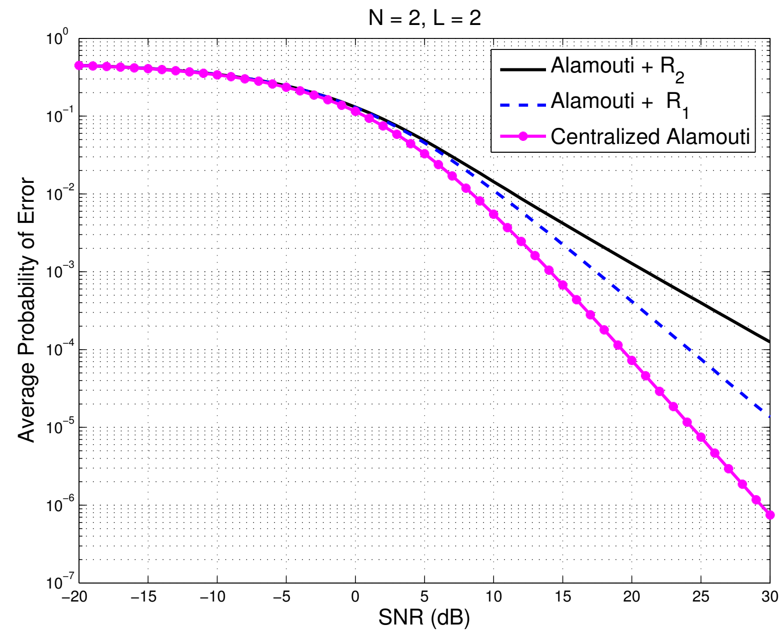
- $\mathbf{r}_i$'s are independent $\Rightarrow$ decentralized
- The received signal at the destination is

$$\mathbf{y} = \boldsymbol{\mathcal{G}}(\mathbf{s})\boldsymbol{\mathcal{R}}\mathbf{h} + \mathbf{w},$$

$\mathbf{w}$ is the AWGN and $\mathbf{h}$ is the channel vector.

Example: randomized Alamouti

- ST code is Alamouti: $P = 2, L = 2, \mathcal{G} = \begin{bmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{bmatrix}$ and $N = 2$.
  - Uniform phase randomization:
    $\mathcal{R}_1 = \begin{bmatrix} e^{j\theta_{11}} & e^{j\theta_{12}} \\ e^{j\theta_{21}} & e^{j\theta_{22}} \end{bmatrix}$, where
    $\theta_{kn} \overset{i.i.d.}{\sim} U(-\pi, \pi)$, $n, k \in \{1, 2\}$.
  - Selection randomization:
    $\mathcal{R}_2 = [\mathbf{r}_1 \ \mathbf{r}_2]$, $\mathbf{r}_i \in \{[1 \ \ 0]^t, [0 \ \ 1]^t\}$
    with equal probability.



- Statistics of $\mathcal{R} \iff$ performance

Design Performance Metric

- **Definition:** The diversity order d^* of a scheme with probability of error $P_e(\text{SNR})$ is defined as

$$d^* = \lim_{\text{SNR}\to\infty} \frac{-\log P_e(\text{SNR})}{\log \text{SNR}}. \quad (1)$$

- Given $\boldsymbol{\mathcal{R}}_{L\times N}$, the randomization matrix $\Rightarrow d^* \leq r \triangleq \min\{L, N\}$
- AIM: To find sufficient conditions such that diversity r is achieved.
- Let $\mathcal{M} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots \mathbf{s}_{|\mathcal{M}|}\} \triangleq$ message set, and $\boldsymbol{\mathcal{G}}_i \triangleq \boldsymbol{\mathcal{G}}(\mathbf{s}_i)$.
- Let $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_N$ be $\text{eig}(\mathbf{A})$, $\mathbf{A} = \mathbf{A}^H$, $|\mathbf{A}|_{k+} = \prod_{\ell=N-k+1}^{N} \lambda_\ell$.

Upper Bound on P_e

Theorem 1 *Assume the scheme satisfies the conditions:*

C1) Rank Criterion for $\mathcal{G}$: $\forall\{\mathcal{G}_k, \mathcal{G}_i\}$, $(\mathcal{G}_k - \mathcal{G}_i)$ is of rank L.

C2) Rank Criterion for $\mathcal{R}$: $\mathcal{R}$ is full-rank with probability 1.

Then, under coherent decoding P_e *is bounded as*

$$P_e \leq \frac{4^{-r}(|\mathcal{M}|-1)\text{SNR}^{-r}}{\min_{(i,j)}\{\,|(\mathcal{G}_i-\mathcal{G}_j)^H(\mathcal{G}_i-\mathcal{G}_j)|_{r+}\}}\mathbb{E}\left\{\frac{1}{|\mathcal{R}\mathcal{R}^H|_{r+}}\right\}. \tag{2}$$

- C1 is equivalent to the rank criterion for deterministic space-time code [Tarokh et al, 1998] $\Rightarrow$ choose $\mathcal{G}$ designed for a multi-antenna system.
- C2 is satisfied if the columns of $\mathcal{R}$ are independently drawn from a continuous distribution.

Finiteness of $\mathbb{E}\left\{1/|\mathcal{R}\mathcal{R}^H|_{r+}\right\}$

Theorem 2 *Let $\mathcal{R}$ be an $L \times N$ random matrix, and $r = \min\{L, N\}$.*

1. Let $p(\mathcal{R})$ be the bounded probability density function of $\mathcal{R}$.

2. Assume that Tr$(\mathcal{R}\mathcal{R}^H) \leq P_T$ with probability 1 (transmission power constraint).

$$|N - L| \geq 1 \Rightarrow \mathbb{E}\{|\mathcal{R}\mathcal{R}^H|_{r+}^{-1}\} < \infty.$$

Definition: A non-negative function $f(\text{SNR}) = \Theta(x^\alpha)$ as $x \to 0$ if $\exists\ \epsilon > 0$ and $0 < c_1 < c_2$ such that $|x| < \epsilon$ implies $c_1 x^\alpha \leq f(\text{SNR}) \leq c_2 x^\alpha$

Lemma 1 $F(x) = \Pr\{|\mathcal{R}\mathcal{R}^H|_{r+} \leq x\}$*: $F(x) = \Theta(x^\alpha)$ around $x = 0$ for some $\alpha \in R$ Then,*

$$\mathbb{E}\{|(\mathcal{R}\mathcal{R}^H)|_{r+}^{-1}\} < \infty \iff \alpha > 1. \quad (3)$$

Diversity order of randomized ST

- MAIN OBSERVATION (Thm. 2): If the rank conditions C1 and C2, $p(\boldsymbol{\mathcal{R}}) < \infty$ and Tr$(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{R}}^H) < \infty$ with probability 1, then

$$d^* = \begin{cases} N & \text{if } L \geq N+1 \\ L & \text{if } L \leq N-1 \end{cases} \quad (4)$$

  For $N = L$, we naturally expect that $N - 1 \leq d^* \leq N$.

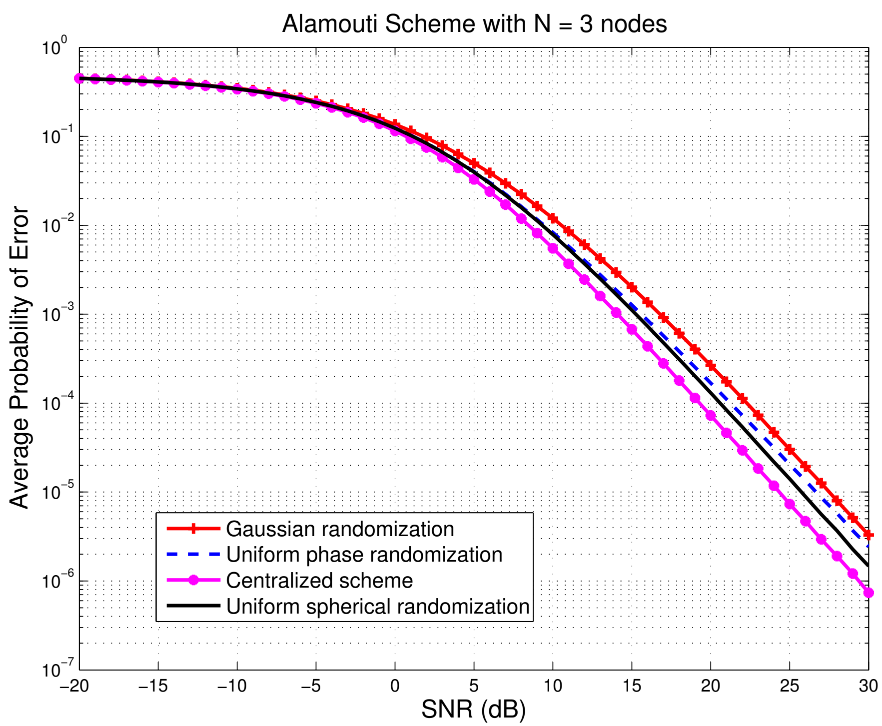
- Remarks:
  - If we choose L large enough, randomized space-time codes achieve diversity order equal to the number of nodes N.
  - It is sufficient to have 1 more node than L to have full diversity L.

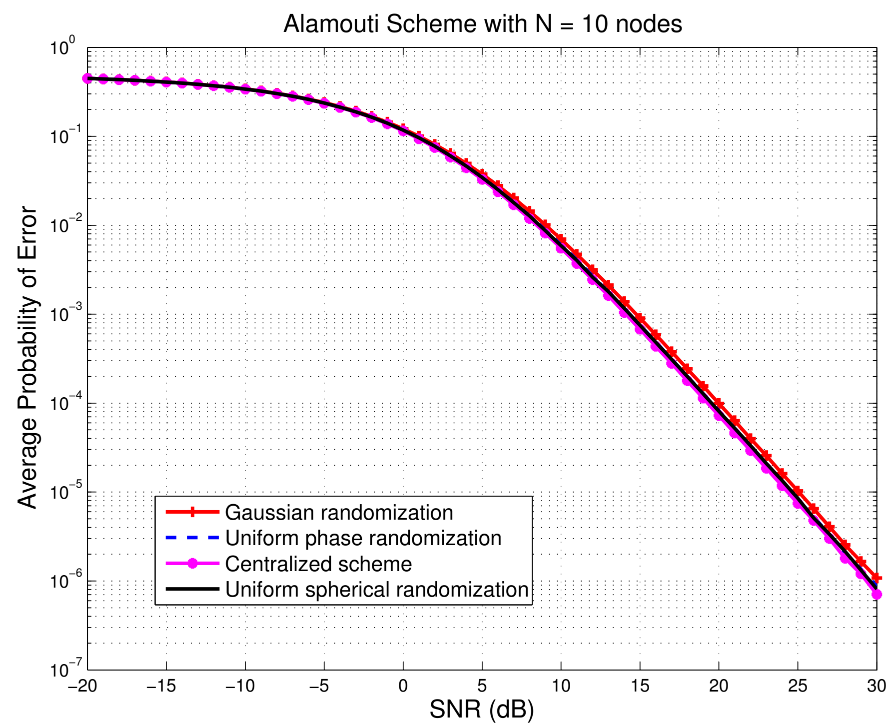
Heuristic Designs

- Let $\mathbf{r}_k$ be the k'th column of the randomization matrix $\mathcal{R}$.
- Gaussian Randomization: The elements of the randomization matrix $\mathcal{R}$ are zero-mean independent and complex Gaussian.
- Uniform Phase Randomization: $\mathbf{r}_k = a_k[e^{j\theta_i[0]}, \ldots, e^{j\theta_i[L]}]^t$ where each $\theta_i[N] \overset{iid}{\sim} U(0, 2\pi)$ and $a_k \overset{iid}{\sim} U(1-\epsilon, 1+\epsilon)$ for some small $\epsilon > 0$, $\theta_i[N]$'s are independent of a_k's.
- Uniform distribution on a complex hypersphere: $\mathbf{r}_k$ is uniformly selected on the surface of a complex hypersphere of radius ρ, *i.e.,* $||\mathbf{r}_k|| = \rho$.

Simulations

- Alamouti scheme with $N = 3$ and $N = 10$





Conclusions

- Randomized ST coding can truly decentralize the use of space-time coding in distributed networks.
- Different designs can provide the diversity order ($\min(N, L)$) when number of nodes N is different than the number of virtual antennas L. For $N = L$, diversity order can be fractional.
- The randomized schemes achieve the performance of a centralized space-time code in terms of coding gain as the number of nodes increases.