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Design of Minimum Error Rate Linear Dispersion Transmission for Cooperative Relay

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Motivation and Introductions

- Enable low-cost, single-antenna terminals to exploit spatial diversity gains offered by Space-Time codes

- Design tradeoff: Allocation of resources between broadcast and cooperative intervals

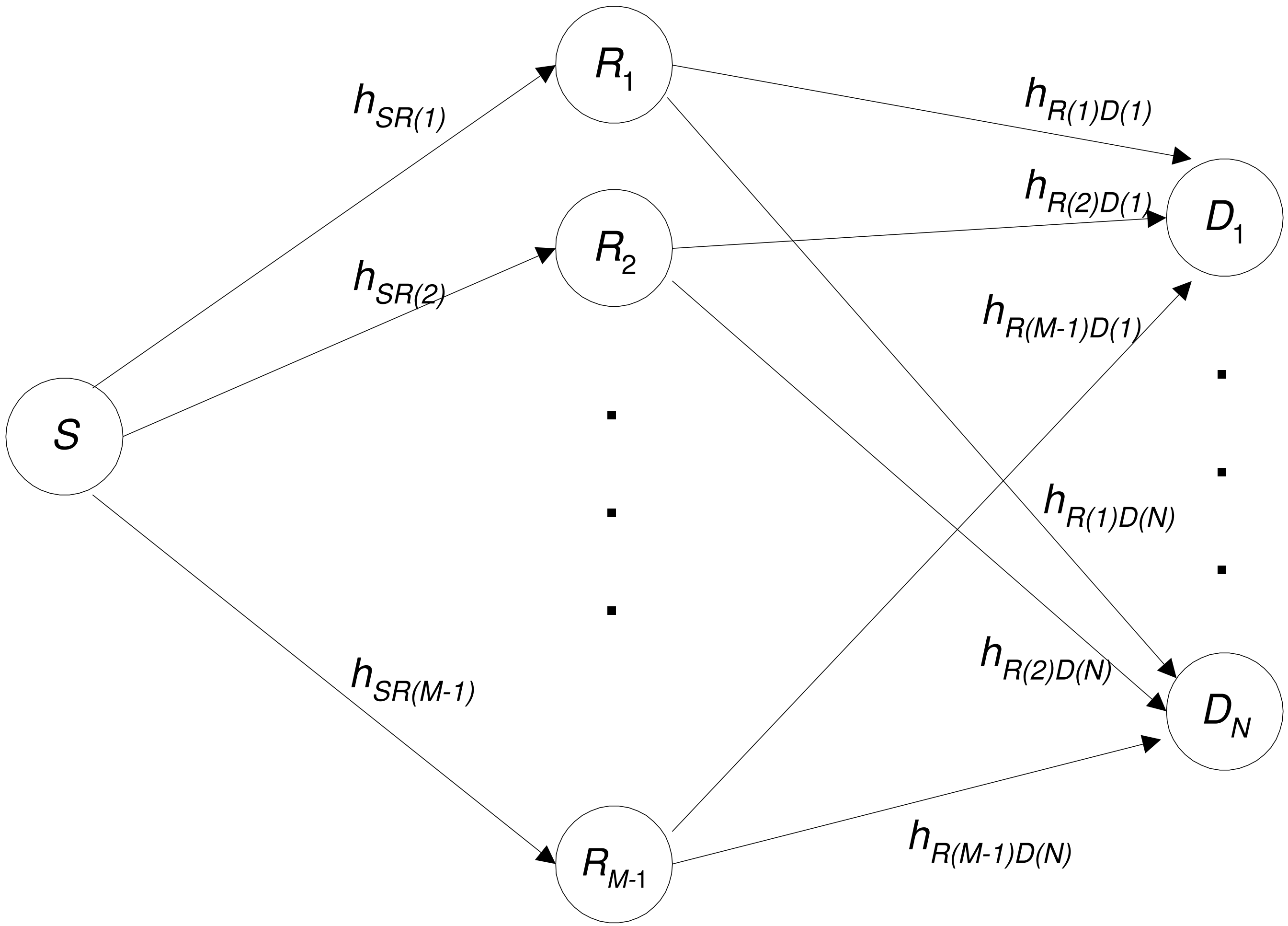
- Simulation-based optimization algorithm to optimize the LD code and power allocation

- Optimal codes for various system configurations and fading statistics

(M, N) System Model

- Terminals: 1 source terminal, $M-1$ relay terminals denoted by S and $R_m, m = 1, \ldots, M-1$, respectively
- Receiver: 1 N-antenna receiver with each antenna denoted by $D_n, n = 1, \ldots, N$
- Channel: source-to-relay, source-to-destination, and relay-to-destination channels are assumed to be mutually independent block-fading channels with arbitrary fading statistics, and are denoted as h_{SR_m}, h_{SD_n}, and $h_{R_mD_n}$, respectively
- Energy Constraint: $E_0 =$ Total energy used

System Model



Transmission Scheme - Broadcast Interval

Define $s_1, s_2, ..., s_Q$ as the Q different r-QAM symbols where $s_q = \alpha_q + j\beta_q$, $q = 1, ..., Q$, and $E\{|s_q^2|\} = 1$

- Energy Constraint: $E_1 = E_0 \cos^2 \alpha, \alpha \in (0, \pi)$

- Source forms τ linearly combined symbols $\boldsymbol{k} = [k_1, ..., k_\tau]^T$ using Q r-QAM symbols, $s_1, ..., s_Q$, and the first-layer dispersion $\tau \times 1$ vectors $\boldsymbol{c}_q$, $\boldsymbol{d}_q$, $q = 1, \ldots, Q$

- Source transmits $k_1, ..., k_\tau$ to the relays and destination during τ consecutive symbol intervals

Transmission Scheme - Cooperative Interval

- Energy Constraint: $E_2 = E_0 \sin^2 \alpha, \alpha \in (0, \pi)$

- Use Amplify-and-Forward Scheme

- Source and relays form the second-layer LD codeword using their received signals corresponding to $k_1, ..., k_\tau$ and with dispersion matrices $\boldsymbol{A}_t$, $\boldsymbol{B}_t$, $t = 1, ..., \tau$, where $\boldsymbol{A}_t$, $\boldsymbol{B}_t$ have dimensions $(T - \tau) \times M$, and each relay uses one column of the dispersion matrices

SNR and Path Loss

Denote the distance between source and relay R_m as d_{SR_m}, $m = 1, ..., M-1$, distance between source and destination as d_{SD}, distance between relay R_m and destination as d_{R_mD}, and ν is the path loss exponent, the SNR for broadcast and cooperative intervals are:

$$\rho_{SD,1} = \frac{E_1}{\tau}\left(\frac{1}{d_{SD}}\right)^{\nu}, \qquad \rho_{SR_m} = \frac{E_1}{\tau}\left(\frac{1}{d_{SR_m}}\right)^{\nu},$$
$$\rho_{SD,2} = \frac{1}{M}\frac{E_2}{T-\tau}\left(\frac{1}{d_{SD}}\right)^{\nu}, \quad \rho_{R_mD} = \frac{1}{M}\frac{E_2}{T-\tau}\left(\frac{1}{d_{R_mD}}\right)^{\nu}.$$

Broadcast Interval

In τ time intervals:

- Broadcast Interval dispersion matrices: $\boldsymbol{c}_q = [c_{1q}, \ldots, c_{\tau q}]^T$ and $\boldsymbol{d}_q = [d_{1q}, \ldots, d_{\tau q}]^T$, $q = 1, \ldots, Q$

- Transmit:

$$\boldsymbol{k} = \sum_{q=1}^{Q} \left(\alpha_q \boldsymbol{c}_q + j\beta_q \boldsymbol{d}_q\right), \quad q = 1, ..., Q$$

- Energy Constraint:

$$\sum_{q=1}^{Q} \left(\boldsymbol{c}_q^H \boldsymbol{c}_q + \boldsymbol{d}_q^H \boldsymbol{d}_q\right) \leq 2\tau$$

Broadcast Interval (cont'd)

Received Signal:

$$\boldsymbol{r}_{R_m} = |h_{SR_m}|\sqrt{\rho_{SR_m}}\boldsymbol{k}+\boldsymbol{n}_{R_m}, \quad \text{with} \quad \boldsymbol{n}_{R_m} \sim \mathcal{N}_{\mathcal{C}}\left(\boldsymbol{0}, \boldsymbol{I}\right), \quad m = 1, \ldots, M-1$$

$$\boldsymbol{r}_{D_n,B} = h_{SD_n}\sqrt{\rho_{SD,1}}\boldsymbol{k}+\boldsymbol{n}_{D_n}, \quad \text{with} \quad \boldsymbol{n}_{D_n} \sim \mathcal{N}_{\mathcal{C}}\left(\boldsymbol{0}, \boldsymbol{I}\right), \quad n = 1, \ldots, N$$

Define:

$$\boldsymbol{y}_1 = \begin{bmatrix} \Re\{\boldsymbol{r}_{R_1}^T\} & \Im\{\boldsymbol{r}_{R_1}^T\} & \cdots & \Re\{\boldsymbol{r}_{R_{M-1}}^T\} & \Im\{\boldsymbol{r}_{R_{M-1}}^T\} \end{bmatrix}^T_{2(M-1)\tau\times 1}$$

which can be written as

$$\boldsymbol{y}_1 = \boldsymbol{H}_1\boldsymbol{G}\boldsymbol{x} + \boldsymbol{n}, \quad \boldsymbol{n} \sim \mathcal{N}\left(\boldsymbol{0}, \frac{1}{2}\boldsymbol{I}\right)$$

Broadcast Interval (cont'd)

$$\begin{bmatrix} \tilde{\alpha}_1 \\ \vdots \\ \tilde{\alpha}_\tau \\ \tilde{\beta}_1 \\ \vdots \\ \tilde{\beta}_\tau \end{bmatrix}_{2\tau\times 1} = \underbrace{\begin{bmatrix} \Re\{c_{11}\} & \cdots & -\Im\{d_{1Q}\} \\ \vdots & \ddots & \vdots \\ \Re\{c_{\tau 1}\} & \cdots & -\Im\{d_{\tau Q}\} \\ \Im\{c_{11}\} & \cdots & \Re\{d_{1Q}\} \\ \vdots & \ddots & \vdots \\ \Im\{c_{\tau 1}\} & \cdots & \Re\{d_{\tau Q}\} \end{bmatrix}_{2\tau\times 2Q}}_{\boldsymbol{G}} \underbrace{\begin{bmatrix} \alpha_1 \\ \beta_1 \\ \vdots \\ \alpha_Q \\ \beta_Q \end{bmatrix}_{2Q\times 1}}_{\boldsymbol{x}}$$

$$\boldsymbol{H}_1 = \begin{bmatrix} |h_{SR_1}|\sqrt{\rho_{SR_1}}\boldsymbol{I}_{2\tau\times 2\tau} \\ \vdots \\ |h_{SR_{M-1}}|\sqrt{\rho_{SR_{M-1}}}\boldsymbol{I}_{2\tau\times 2\tau} \end{bmatrix}_{2(M-1)\tau\times 2\tau}$$

Cooperative Interval

In $T - \tau$ time intervals:

- Cooperative Interval dispersion matrices: $\{\boldsymbol{A}_t, \boldsymbol{B}_t\}_{t=1}^{\tau}$ with constraint

$$\sum_{t=1}^{\tau} \mathrm{tr}\left(\boldsymbol{A}_t^H \boldsymbol{A}_t + \boldsymbol{B}_t^H \boldsymbol{B}_t\right) \leq 2M(T - \tau)$$

- Normalize received signal from Broadcast interval:

$$\gamma_{R_m} = \sqrt{\frac{\tau}{|h_{SR_m}|^2 \rho_{SR_m} \tau + \tau}}, \quad m = 1, \ldots, M-1$$

$$\gamma_{R_m}^2 E\{\boldsymbol{r}_{R_m}^H \boldsymbol{r}_{R_m}\} = \tau$$

Cooperative Interval (cont'd)

- Transmitted signal:

$$\begin{aligned} \boldsymbol{x}_S &= \sum_{t=1}^{\tau} \Re\{k_t\}\boldsymbol{a}_{1,t} + j\Im\{k_t\}\boldsymbol{b}_{1,t} \\ \boldsymbol{x}_{R_m} &= \gamma_{R_m} \sum_{t=1}^{\tau} \Big(\Re\{r_{R_m,t}\}\boldsymbol{a}_{m+1,t} + j\Im\{r_{R_m,t}\}\boldsymbol{b}_{m+1,t}\Big), \\ & \quad m = 1, \ldots, M-1 \end{aligned}$$

- Received Signal:

$$\begin{aligned} \boldsymbol{r}_{D_n,C} &= h_{SD_n}\sqrt{\rho_{SD,2}}\boldsymbol{x}_S + \sum_{m=1}^{M-1} h_{R_mD_n}\sqrt{\rho_{R_mD_n}}\boldsymbol{x}_{R_m} + \boldsymbol{v}_{D_n}, \\ & \quad \boldsymbol{v}_{D_n} \sim \mathcal{N}_{\mathcal{C}}(\boldsymbol{0}, \boldsymbol{I}) \end{aligned}$$

Cooperative Interval (cont'd)

Define:

$$\boldsymbol{y}_2 = \begin{bmatrix} \Re\{\boldsymbol{r}_{D_1,B}^T\} & \Im\{\boldsymbol{r}_{D_1,B}^T\} & \cdots & \Re\{\boldsymbol{r}_{D_N,B}^T\} & \Im\{\boldsymbol{r}_{D_N,B}^T\} \\ \Re\{\boldsymbol{r}_{D_1,C}^T\} & \Im\{\boldsymbol{r}_{D_1,C}^T\} & \cdots & \Re\{\boldsymbol{r}_{D_N,C}^T\} & \Im\{\boldsymbol{r}_{D_N,C}^T\} \end{bmatrix}_{2NT\times 1}^T$$

We have:

$$\boldsymbol{y}_2 = \boldsymbol{H}_2\boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}_3\boldsymbol{y}_1 + \boldsymbol{u}, \quad \boldsymbol{u} \sim \mathcal{N}\left(\boldsymbol{0}, \frac{1}{2}\boldsymbol{I}\right)$$

or,

$$\boldsymbol{y}_2 = (\boldsymbol{H}_2 + \boldsymbol{H}_3\boldsymbol{H}_1)\,\boldsymbol{G}\boldsymbol{x} + (\boldsymbol{H}_3\boldsymbol{n} + \boldsymbol{u})$$

Cooperative Interval (cont'd)

$$\mathcal{A}_{m,t} = \begin{bmatrix} \Re\{\boldsymbol{a}_{m,t}\} & -\Im\{\boldsymbol{a}_{m,t}\} \\ \Im\{\boldsymbol{a}_{m,t}\} & \Re\{\boldsymbol{a}_{m,t}\} \end{bmatrix}, \quad \mathcal{B}_{m,t} = \begin{bmatrix} -\Im\{\boldsymbol{b}_{m,t}\} & -\Re\{\boldsymbol{b}_{m,t}\} \\ \Re\{\boldsymbol{b}_{m,t}\} & -\Im\{\boldsymbol{b}_{m,t}\} \end{bmatrix}$$

$$\underline{\boldsymbol{h}}_{SD_n} = \sqrt{\rho_{SD,2}} \begin{bmatrix} \Re\{h_{SD_n}\} \\ \Im\{h_{SD_n}\} \end{bmatrix}, \quad \underline{\boldsymbol{h}}_{R_m D_n} = \gamma_{R_m} \sqrt{\rho_{RD}} \begin{bmatrix} \Re\{h_{R_m D_n}\} \\ \Im\{h_{R_m D_n}\} \end{bmatrix}$$

$$\boldsymbol{P} = \sqrt{\rho_{SD,1}} \begin{bmatrix} \boldsymbol{P}_1 & \boldsymbol{P}_2 & \cdots & \boldsymbol{P}_N \end{bmatrix}^T_{2N\tau \times 2\tau}$$

$$\boldsymbol{P}_n = \begin{bmatrix} \Re\{h_{SD_n}\}\boldsymbol{I}_{\tau\times\tau} & \Im\{h_{SD_n}\}\boldsymbol{I}_{\tau\times\tau} \\ -\Im\{h_{SD_n}\}\boldsymbol{I}_{\tau\times\tau} & \Re\{h_{SD_n}\}\boldsymbol{I}_{\tau\times\tau} \end{bmatrix}_{2\tau\times 2\tau}$$

Cooperative Interval (cont'd)

$$\boldsymbol{H}_2 \begin{bmatrix} & & \boldsymbol{P} & & & \\ \mathcal{A}_{1,1}\underline{\boldsymbol{h}}_{SD_1} & \cdots & \mathcal{A}_{1,\tau}\underline{\boldsymbol{h}}_{SD_1} & \mathcal{B}_{1,1}\underline{\boldsymbol{h}}_{SD_1} & \cdots & \mathcal{B}_{1,\tau}\underline{\boldsymbol{h}}_{SD_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{A}_{1,1}\underline{\boldsymbol{h}}_{SD_N} & \cdots & \mathcal{A}_{1,\tau}\underline{\boldsymbol{h}}_{SD_N} & \mathcal{B}_{1,1}\underline{\boldsymbol{h}}_{SD_N} & \cdots & \mathcal{B}_{1,\tau}\underline{\boldsymbol{h}}_{SD_N} \end{bmatrix}$$

$$\boldsymbol{H}_3 \begin{bmatrix} & & & \boldsymbol{0}_{2N\tau\times 2(M-1)\tau} & & \\ \mathcal{A}_{2,1}\underline{\boldsymbol{h}}_{R_1D_1} & \cdots & \mathcal{A}_{2,\tau}\underline{\boldsymbol{h}}_{R_1D_1} & \mathcal{B}_{2,1}\underline{\boldsymbol{h}}_{R_1D_1} & \cdots & \mathcal{B}_{M,\tau}\underline{\boldsymbol{h}}_{R_{M-1}D_1} \\ \mathcal{A}_{2,1}\underline{\boldsymbol{h}}_{R_1D_2} & \cdots & \mathcal{A}_{2,\tau}\underline{\boldsymbol{h}}_{R_1D_2} & \mathcal{B}_{2,1}\underline{\boldsymbol{h}}_{R_1D_2} & \cdots & \mathcal{B}_{M,\tau}\underline{\boldsymbol{h}}_{R_{M-1}D_2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{A}_{2,1}\underline{\boldsymbol{h}}_{R_1D_N} & \cdots & \mathcal{A}_{2,\tau}\underline{\boldsymbol{h}}_{R_1D_N} & \mathcal{B}_{2,1}\underline{\boldsymbol{h}}_{R_1D_N} & \cdots & \mathcal{B}_{M,\tau}\underline{\boldsymbol{h}}_{R_{M-1}D_N} \end{bmatrix}$$

Optimization of Two-Layer Cooperative LD Code

Define:

$$\boldsymbol{\theta} = \Big\{ \{\boldsymbol{c}_q, \boldsymbol{d}_q, q = 1, \ldots, Q\}, \quad \{\boldsymbol{A}_t, \boldsymbol{B}_t, t = 1, \ldots, \tau\}, \quad \alpha \Big\}$$

Define the average empirical BLER as:

$$\Upsilon(\boldsymbol{\theta}) = E_{\boldsymbol{x}} E_{\boldsymbol{h}_1} E_{\boldsymbol{h}_2} E_{\boldsymbol{y}_1, \boldsymbol{y}_2 | \boldsymbol{x}, \boldsymbol{h}_1, \boldsymbol{h}_2} \{\gamma(\boldsymbol{y}_2, \boldsymbol{x}, \boldsymbol{h}_1, \boldsymbol{h}_2, \boldsymbol{\theta})\}$$

Solve the optimization problem:

$$\min_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} \Upsilon(\boldsymbol{\theta})$$

with the constraint set given by

$$\begin{aligned} \boldsymbol{\Theta} \; = \; & \Big\{ \sum_{q=1}^{Q} \mathrm{tr}\Big(\boldsymbol{c}_q^H \boldsymbol{c}_q + \boldsymbol{d}_q^H \boldsymbol{d}_q\Big) \leq 2\tau, \\ & \sum_{t=1}^{\tau} \mathrm{tr}\Big(\boldsymbol{A}_t^H \boldsymbol{A}_t + \boldsymbol{B}_t^H \boldsymbol{B}_t\Big) \leq 2M(T - \tau) \Big\} \end{aligned}$$

Simulation-Based LD Code Optimization Algorithm

1. Generate symbol and signal samples:

   1) Draw L symbol vectors $\boldsymbol{x}(1)$, $\boldsymbol{x}(2)$, . . . , $\boldsymbol{x}(L)$, uniformly from the constellation set

   2) Simulate L observations $\boldsymbol{y}_1(1)$, $\boldsymbol{y}_1(2)$, . . . , $\boldsymbol{y}_1(L)$

   $$\boldsymbol{y}_1(\ell) = \boldsymbol{H}_1(\ell)\boldsymbol{G}(\ell)\boldsymbol{x}(\ell) + \boldsymbol{n}(\ell), \quad \ell = 1, 2, \ldots, L$$

   3) Simulate L observations $\boldsymbol{y}_2(1)$, $\boldsymbol{y}_2(2)$, . . . , $\boldsymbol{y}_2(L)$

   $$\boldsymbol{y}_2(\ell) = \boldsymbol{H}_2(\ell)\boldsymbol{G}(\ell)\boldsymbol{x}(\ell) + \boldsymbol{H}_3(\ell)\boldsymbol{y}_1(\ell) + \boldsymbol{u}(\ell), \quad \ell = 1, 2, ..., L$$

   4) Decode $\boldsymbol{x}(\ell)$ and compute the empirical BLER $\gamma\Big(\boldsymbol{z}(\ell), \boldsymbol{x}(\ell), \boldsymbol{h}_1(\ell), \boldsymbol{h}_2(\ell), \boldsymbol{\theta}_k\Big)$

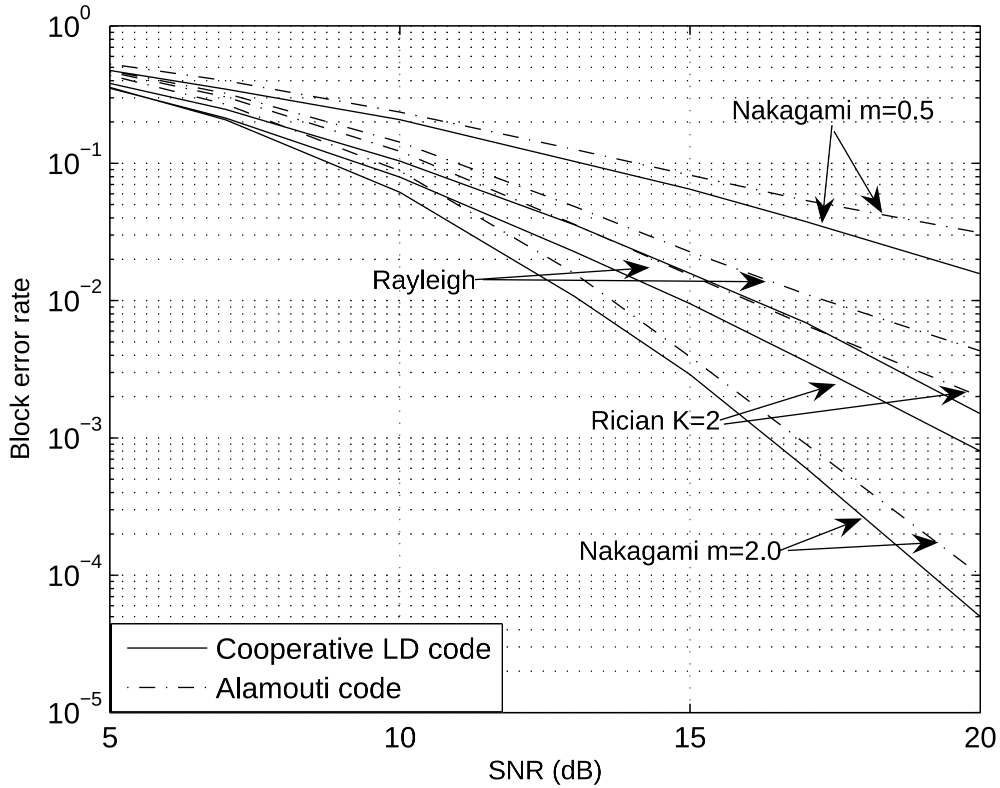
2. Score function method for gradient estimation: Generate the estimate

$$\begin{aligned}\widehat{\boldsymbol{g}}\left(\boldsymbol{\theta}_k\right) &= \frac{1}{L}\sum_{\ell=1}^{L}\gamma\Big(\boldsymbol{y}_2(\ell),\boldsymbol{x}(\ell),\boldsymbol{h}_1(\ell),\boldsymbol{h}_2(\ell),\boldsymbol{\theta}_k\Big)\\ &\quad\times\Big\{\nabla_{\boldsymbol{\theta}}\Big[\log p\left(\boldsymbol{y}_1(\ell)\,|\,\boldsymbol{x}(\ell),\boldsymbol{h}_1(\ell),\boldsymbol{\theta}\right)\\ &\quad+\log p\left(\boldsymbol{y}_2(\ell)\,|\,\boldsymbol{y}_1(\ell),\boldsymbol{x}(\ell),\boldsymbol{h}_2(\ell),\boldsymbol{\theta}\right)\Big]\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_k}\Big\}\end{aligned}$$

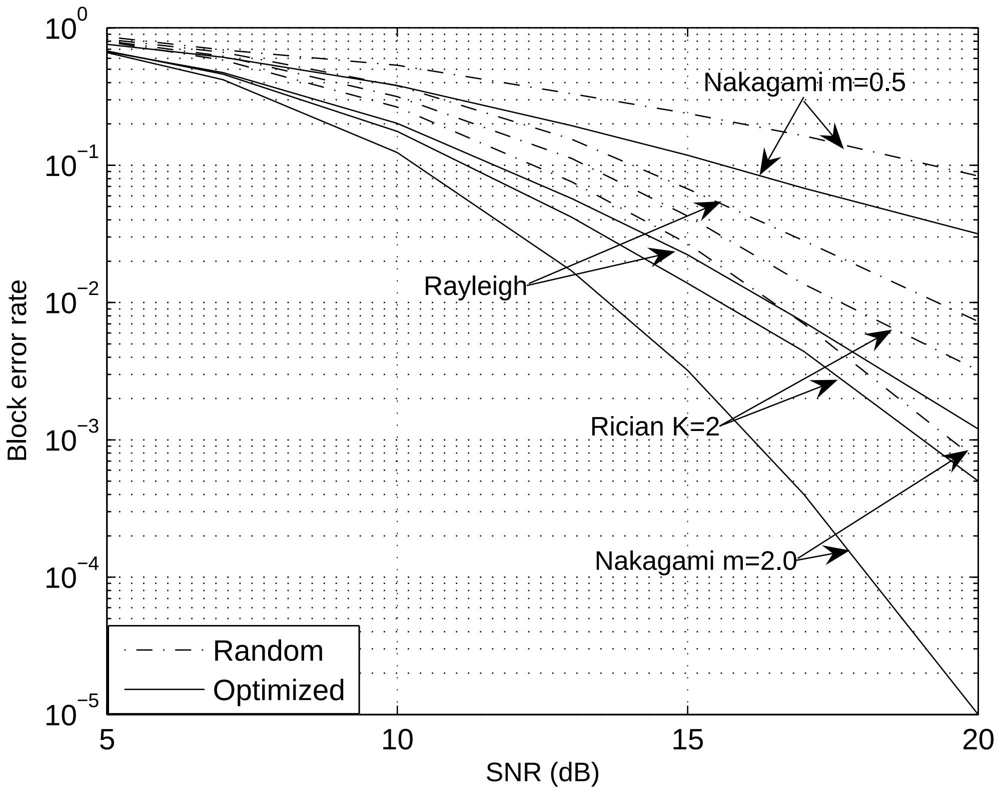
3. Update parameters: The parameters are updated as

$$\boldsymbol{\theta}_{k+1} = \Pi_{\boldsymbol{\Theta}}\left[\boldsymbol{\theta}_k - a_k\widehat{\boldsymbol{g}}(\boldsymbol{\theta}_k)\right]$$

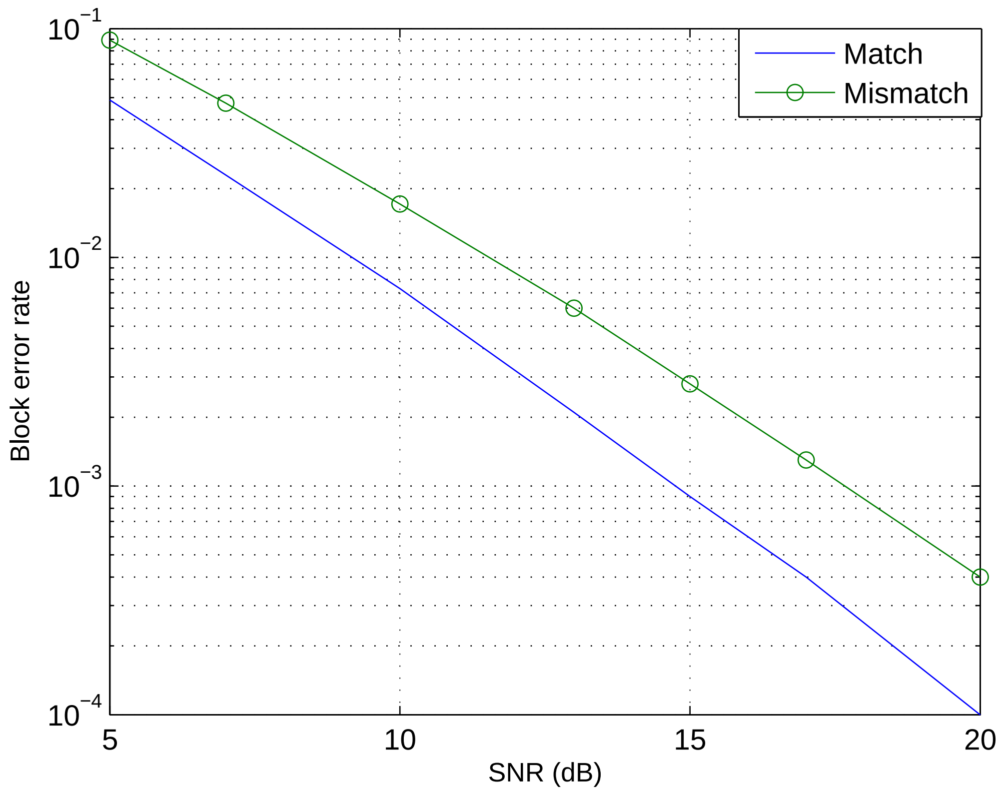
where $\Pi_{\boldsymbol{\Theta}}(\cdot)$ is a projection operator onto the set $\boldsymbol{\Theta}$



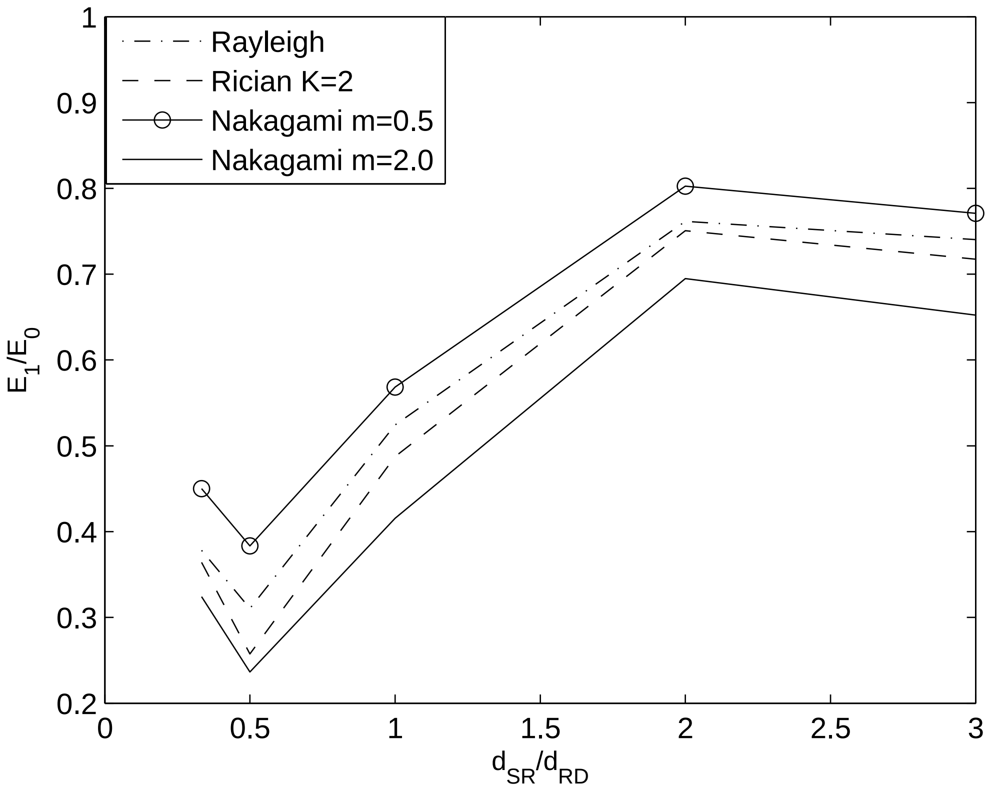
Cooperative LD code vs Alamouti code for system with 1 source, 1 relay, and 2 destination receive antenna.



Optimized cooperative LD code vs randomly chosen cooperative LD code for system with 1 source, 3 relay2, and 2 destination receive antenna.



Cooperative LD code optimized for different terminal distances.



Ratio of energy allocation in broadcast interval for different D_{sr}/D_{rd} ratios.