Design of LDPC Codes for Cooperative Diversity Systems

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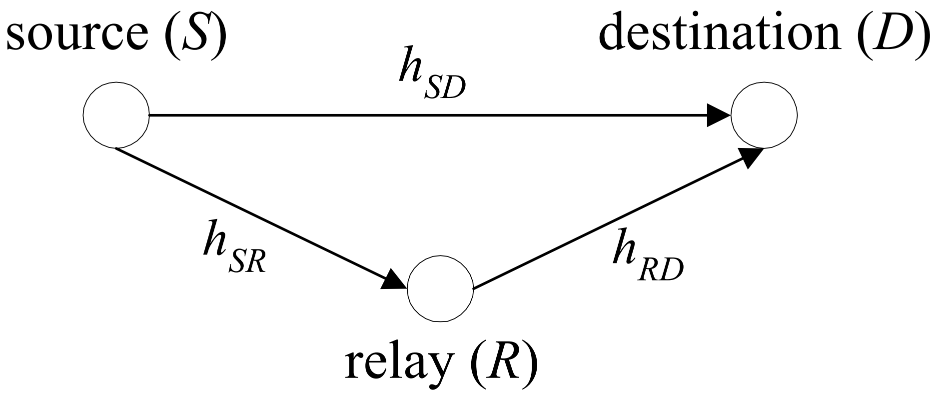
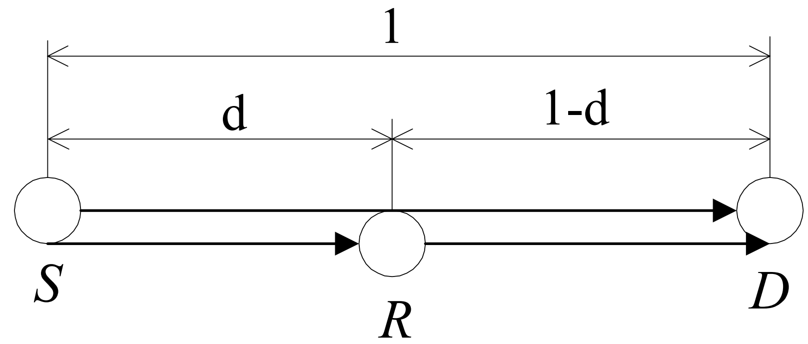
Outline

- Background
- LDPC code design for cooperative relay systems
- Simulation results
- Conclusions

Background: Cooperative Relay System



(a) diagram of a single-relay system

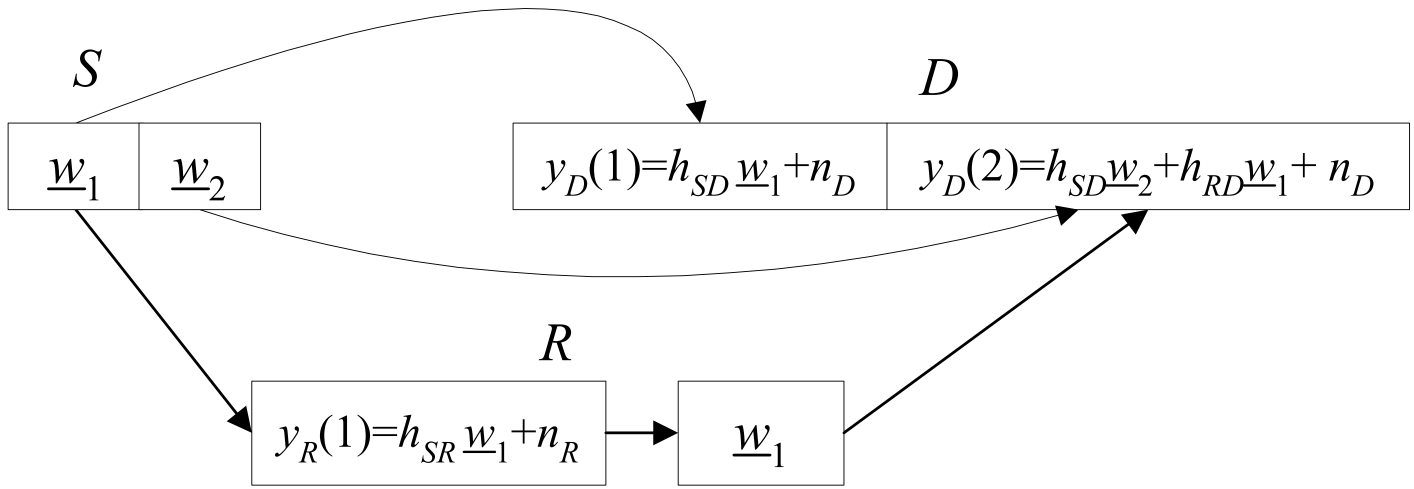


(b) one-dim channel model

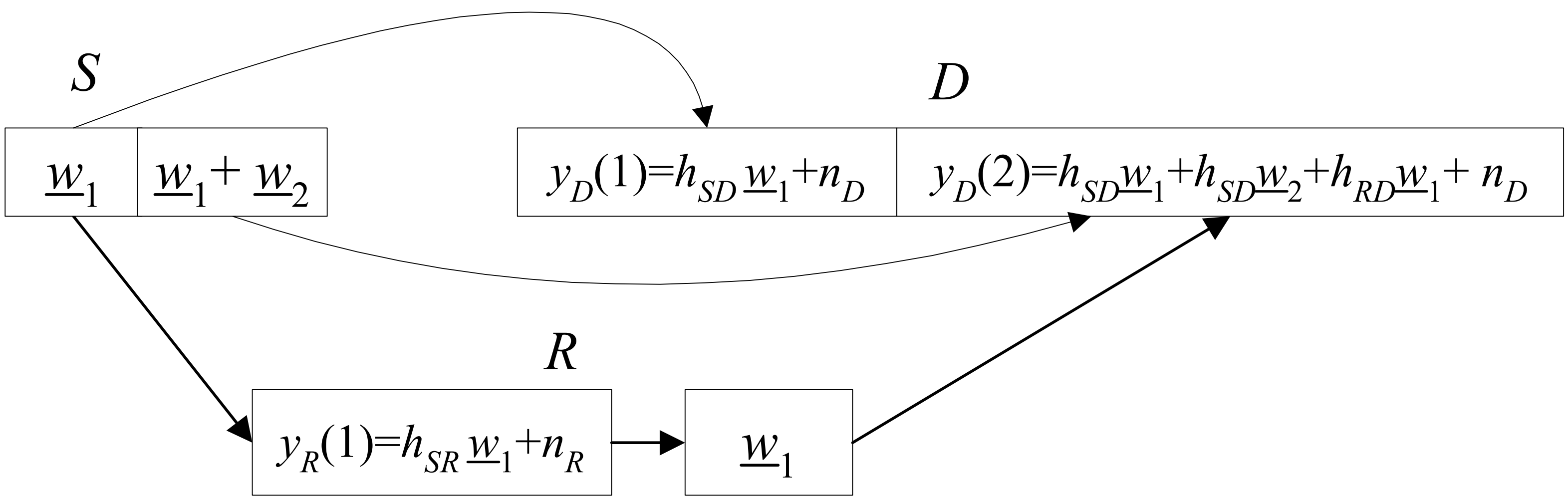
- <u>Relay node operations:</u>
  - receive the signal from *S*;
  - amplify or decode the signal;
  - transmit the signal to *D*.
- <u>Destination node operations:</u>

  receive and decode the signals from both *S* and *R*

Background: Two Relay Protocols



Simple protocol



"Decode-and-forward" (DF) protocol

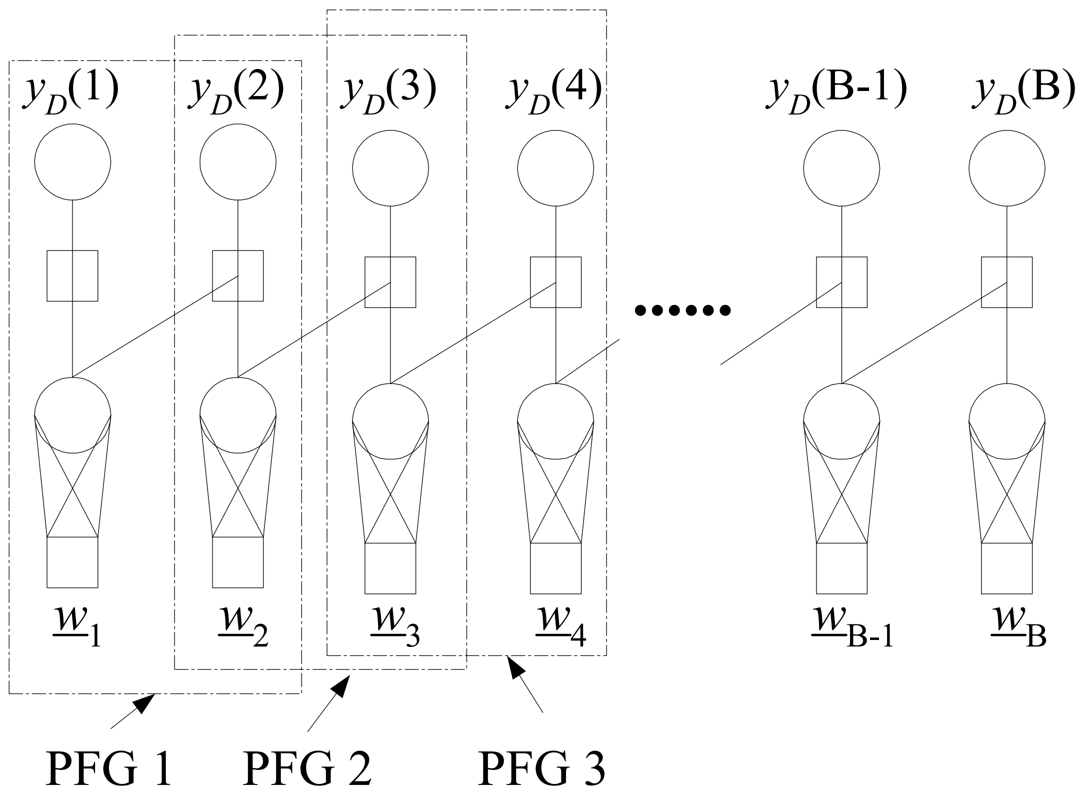
Background: Factor Graph (FG)

- <u>Factor graph representation:</u>

  entire FG (B-block) and partial FGs (PFG, 2-block)

- <u>Optimum code design:</u> over the entire FG

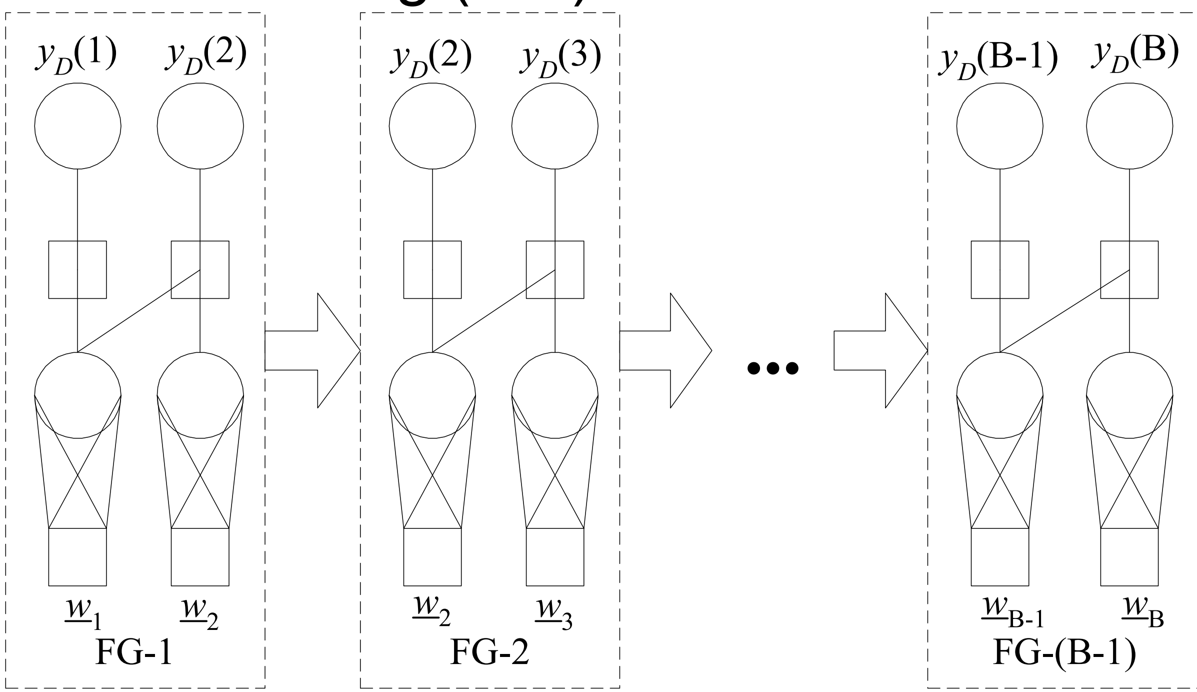
  high computational complexity and long delay in decoding



Background: FG Decoupling

- <u>Suboptimum code design:</u>

  over the PFG (instead of the entire FG)
- <u>Successive decoding:</u>
  - corresponding to the structure of successive PFGs
  - forward-decoding (FW) and backward-decoding (BW)



Code Design: Relay Operations

- Simple protocol: AWGN $y_R(t) = \sqrt{P_S} h_{SR} \underline{w}_t + n_R$
- DF protocol: virtual MISO model

$$y_R(t) = \sqrt{P_{S,1}} h_{SR} \underline{w}_t + \sqrt{P_{S,2}} h_{SR} \underline{w}_{t+1} + n_R$$

i.e., $y_R(t) = h_{SR}[\sqrt{P_{S,1}}, \sqrt{P_{S,2}}][\underline{w}_t, \underline{w}_{t+1}]^T + n_R$

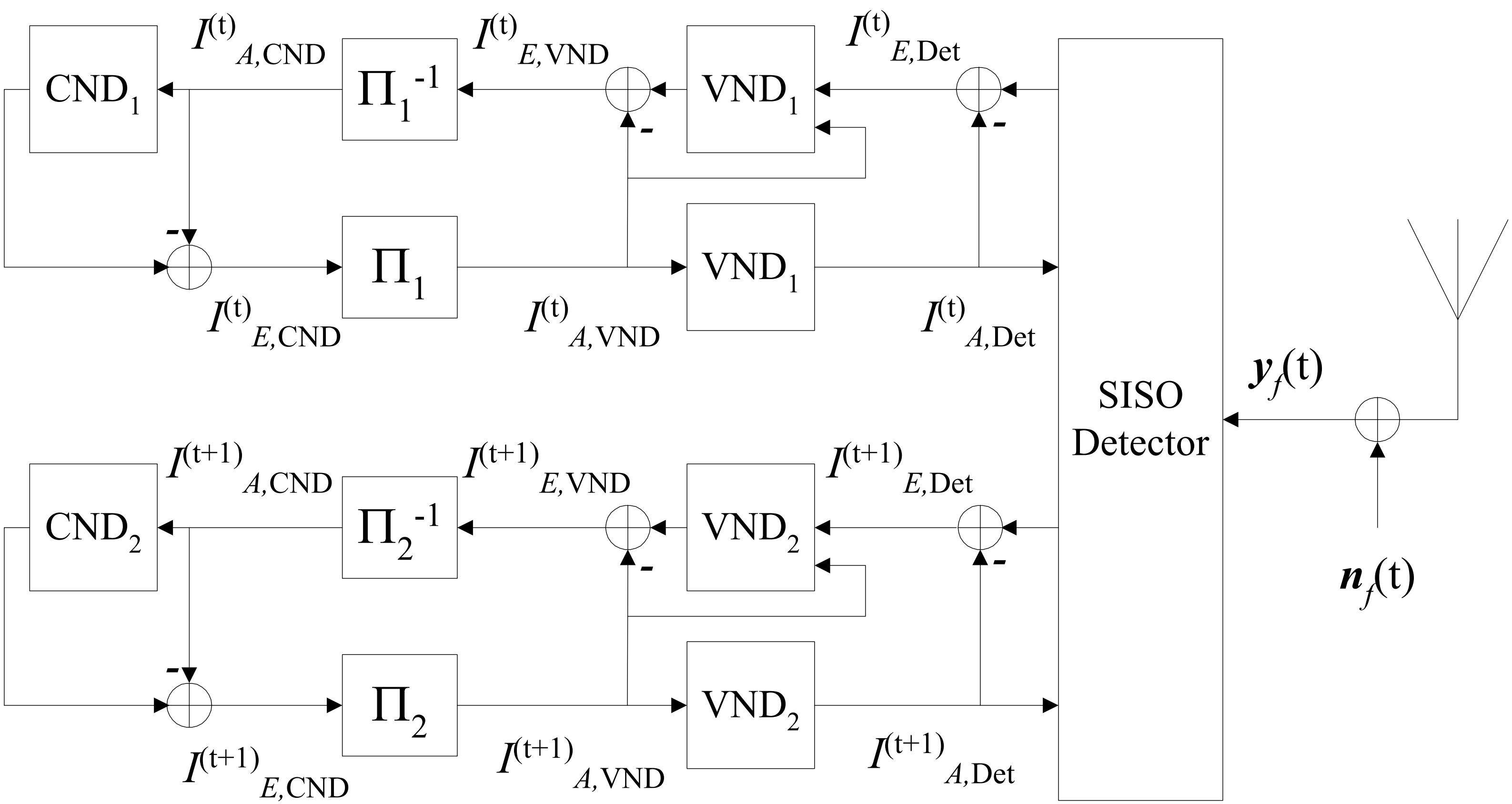
- EXIT chart analysis for LDPC-coded relay (virtual MISO)

$$I_R(t) \text{ and } \sigma_R(t) = J^{-1}(I_R(t))$$

  - Two independent LDPC decoders: w_t and w_{t+1} (or w_{t-1})
  - The relay output is the decoder output for $\widehat{w}_t$
  - The decoding of w_{t+1} only helps to improve the *prior* in the decoding of $\widehat{w}_t$
  - The relay output $I_R(t)$ will be exploited as the *prior* during the time slot (*t*+1) when decoding $\widehat{w}_{t+1}$

Code Design: Relay Performance Analysis

- Iterative receiver for the LDPC-coded relay node



Code Design: Destination Operations (Virtual MIMO Model)

$$y(t) = Hx(t) + n(t),$$

$$y(t) = [y_D(t), y_D(t+1)]^T \quad , x(t) = [w(t), \hat{w}(t), w(t+1), \hat{w}(t+1)]^T$$

- Simple protocol: MIMO
  - FW-dec.: $$H_S^f = \begin{bmatrix} h_{SD}\sqrt{P_S} & 0 & 0 & 0 \\ 0 & h_{RD}\sqrt{P_R} & 0 & h_{SD}\sqrt{P_S} \end{bmatrix}$$
  - BW-dec.: $$H_S^b = \begin{bmatrix} 0 & h_{RD}\sqrt{P_R} & 0 & 0 \\ h_{SD}\sqrt{P_S} & 0 & 0 & h_{RD}\sqrt{P_R} \end{bmatrix}$$
- DF protocol: MIMO
  - FW-dec.: $$H_{DF}^f = \begin{bmatrix} h_{SD}\sqrt{P_{S,1}} & 0 & 0 & 0 \\ h_{SD}\sqrt{P_{S,.2}} & h_{RD}\sqrt{P_R} & h_{SD}\sqrt{P_{S,1}} & 0 \end{bmatrix}$$
  - BW-dec.: $$H_{DF}^b = \begin{bmatrix} h_{SD}\sqrt{P_{S,2}} & h_{RD}\sqrt{P_R} & 0 & 0 \\ h_{SD}\sqrt{P_{S,1}} & 0 & h_{SD}\sqrt{P_{S,2}} & h_{RD}\sqrt{P_R} \end{bmatrix}$$

Code Design: Destination Performance Analysis

- EXIT analysis for the LDPC-coded destination

$$I_D(t) = \sum_i \lambda_i J(\sqrt{[J^{-1}(I_{E,\text{Det}}(i))]^2 + i[J^{-1}(I_{E,\text{VND}})]^2})$$

  - Two independent LDPC decoders: w_t and w_{t+1} (or w_{t-1})
  - The destination output is the decoder output for $\widehat{w}_t$
  - The decoding of w_{t+1} only helps to improve the *prior* in the decoding of $\widehat{w}_t$
  - The relay output $I_D(t)$ will be exploited as the *prior* in the PFG-(*t*+1) when decoding $\widehat{w}_{t+1}$

Similar receiver structure as that for the relay

Code Design: Conventional Code Performance Analysis

- Relay decoding to get $\widehat{w}_t$ for each w_t

  The imperfect relaying effects are ***numerically*** incorporated into the destination performance via the virtual MIMO model.

  $$y = Hx + n \text{ where } x = [w_t, \widehat{w}_t, w_{t+1}, \widehat{w}_{t+1}]$$

- The destination detector output $I_{E,\text{Det}}(i)$ within each iteration

  $$\text{ML detection : } L_{E,\text{Det}} = \log \frac{\sum_{w_{t+1}} \sum_{\widehat{w}_t} p(y|w_t{=}1)}{\sum_{w_{t+1}} \sum_{\widehat{w}_t} p(y|w_t{=}-1)}$$

  $$\underline{Numerically} \text{ obtained function : } I_{E,\text{Det}}(i) = f_{\text{Det}}(I_{A,\text{Det}}(i), \tfrac{E_b}{N_0})$$

  All ***numerical*** procedures, high computational complexity and thus slow evolution

Code Design: Semi-analytical Performance Analysis Approaches

- BSC approximation for the relay:

$$\hat{w}_t = f_{\mathsf{R}}(w_t) = w_t \text{ or } -w_t$$

with the crossover probability $P_0 = P(\hat{w}_t \neq w_t | w_t) = Q(\frac{J^{-1}(I_R)}{2})$

  – Approximate the relay output using the BSC output $\hat{w}_t$ (instead of numerically decoding to get $\hat{w}_t$)

- Efficient destination detector with Gaussian approx.:
  – Computationally efficient detection (instead of ML) ***Semi-analytical*** expressions are available for $L_{E,\mathrm{Det}}$.
  – $I_{E,\mathrm{Det}}(i)$ also has ***semi-analytical*** expressions including the parameter P_0 (instead of numerically calculation)
  – The imperfect relaying effects are ***analytically*** incorporated into the destination performance via P_0

Results: Destination Performances (1)

	Protocol	*d* (*S*,*R*)	Dec.	Capacity	Opt. Code	(3,6) Code
Dest. Perf.	Simple	0.25	FW		-1.25dB	-0.30dB
			BW		-1.55dB	-0.75dB
		0.50	FW		-2.50dB	-1.50dB
			BW		-2.90dB	-2.10dB
	DF	0.25	FW	-3.50dB	-3.45dB (**0.05dB** gap)	-2.61dB
			BW	-3.50dB	-3.47dB (**0.03dB** gap)	-2.64dB
		0.50	FW	-3.70dB	-3.25dB	-2.34dB
			BW	-3.70dB	-3.26dB	-2.37dB

Results: Destination Performances (2)

- Significant gain over regular codes:
  - Optimized codes versus regular codes (0.8dB gain)
- Capacity-approaching performance:
  - Optimized codes versus capacity (within 0.1dB gap) (DF protocol, d=0.25)
- Successive decoding schemes:
  - BW-dec. outperforms FW-dec. (0.3~0.4dB gain) BW versus FW (simple protocol)
  - BW-dec. has approximated perf. as FW-dec. BW versus FW (DF protocol)

Imperfect relaying effects!

Results: Relay Performances (1)

	Protocol	$d\ (S,R)$	Dest. perf.	Opt. Codes	(3,6) Code
Relay Perf.	simple	0.25	-1.25dB (FW) -1.55dB (BW)	-4.98dB	-4.17dB
		0.50	-2.50dB (FW) -2.90dB (BW)	-3.26dB	-2.40dB
	DF	0.25	-3.45dB (FW) -3.47dB (BW)	-3.45dB	-2.62dB
		0.50	-3.25dB (FW) -3.26dB (BW)	-3.23dB	-2.35dB

Results: Relay Performances (2)

- Significant gain over regular codes
  - Optimized codes versus regular codes (0.8dB gain)
  - Note: The optimized codes are designed for the destination (instead of the relay).
- Perfect relaying
  - Relay perf. versus Dest. perf. (simple protocol)
  - Destination perf.: ***BW outperforms FW*** (over 0.3dB gain)
- Imperfect relaying
  - Relay perf. versus Dest. perf. (DF protocol)
  - Destination perf.: ***BW has approximated perf. as FW***
  - The destination has approximated perf. as the relay.

Relay performances are crucial to the relay system!

Conclusions

- The optimization framework for the LDPC-coded cooperative relay systems
- Efficient approaches for code performance analysis
- The optimized codes can achieve significant gains over the regular codes;
- Under the DF protocol, the optimized codes can approach within 0.1dB gap to the capacity;
- Relay performances are crucial to the system performances.