Codes for Node-Constrained Relaying

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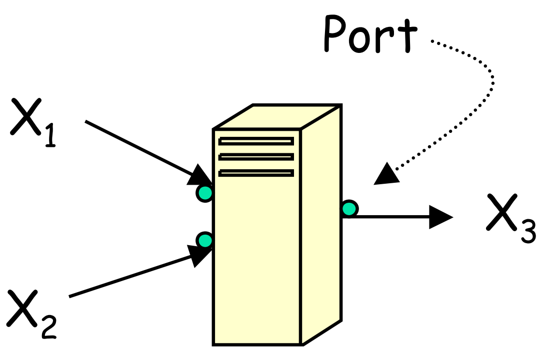
Outline

1) Network nodes
2) Wireline relaying
   - one path: how data compression helps
3) Generalizations and Problems
   - one relay, two paths
   - two relays, two paths
   - many sources/paths, unicast/multicast

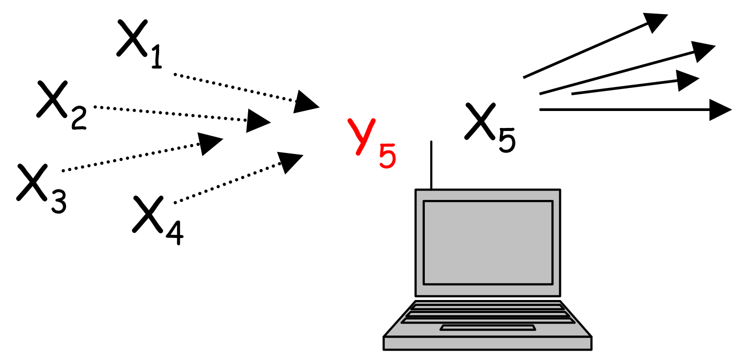
1) Network Nodes

- Wireline



Node constraints:
Suppose k ports can be active at once, e.g., k=1

- Wireless

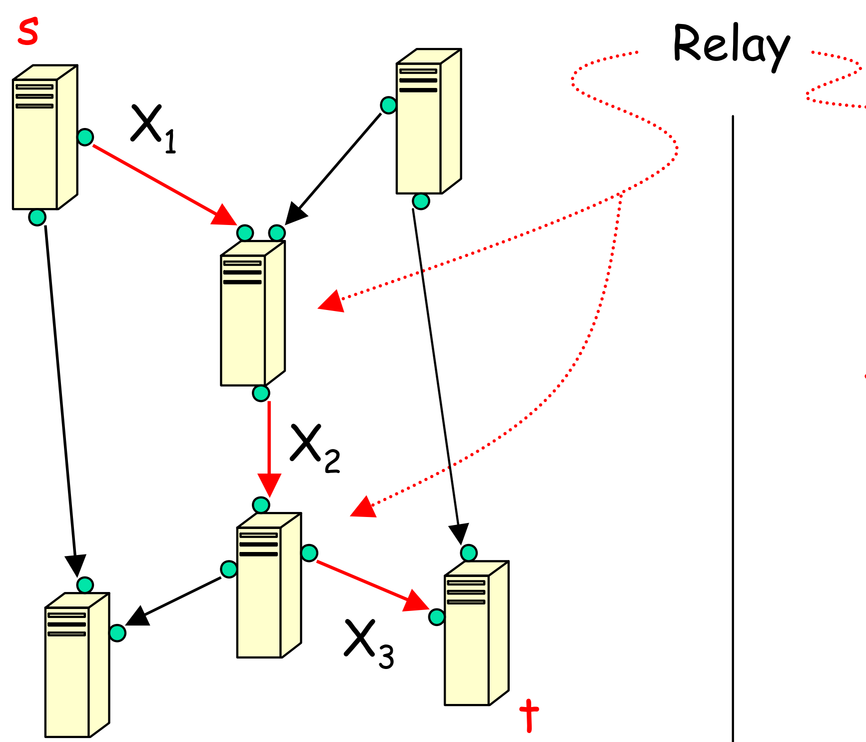


Half-duplex constraint:

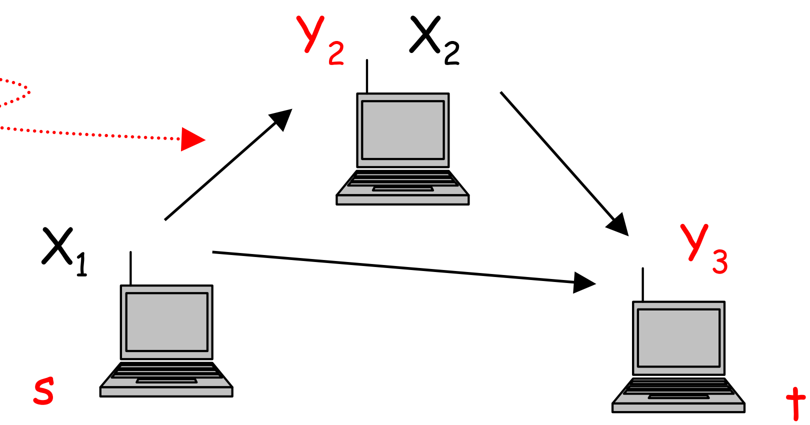
$$Y_t = \begin{cases} Z_t + \sum_{s \neq t} \frac{h_{st}}{d_{st}^{\alpha/2}} X_s & \text{if } X_t = 0 \\ 0 & \text{else} \end{cases}$$

Networks

- Wireline



- Wireless

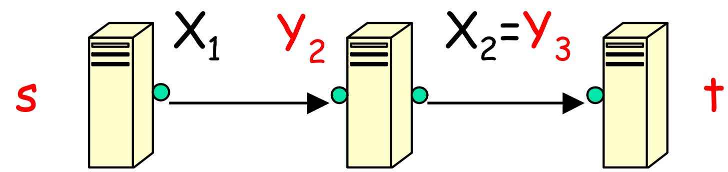


$$Y_2 = \begin{cases} Z_2 + \dfrac{h_{12}}{d_{12}^{\alpha/2}} X_1 & \text{if } X_2 = 0 \\ 0 & \text{else} \end{cases}$$

$$Y_3 = Z_3 + \frac{h_{13}}{d_{13}^{\alpha/2}} X_1 + \frac{h_{23}}{d_{23}^{\alpha/2}} X_2$$

2) Wireline Relaying

- 3 node example:



- Suppose 1 port/node can be active simultaneously. A link (channel) model:

$$\text{if } X_2=0 \text{ then } Y_2=X_1$$
$$\text{if } X_2\neq 0 \text{ then } Y_2=0$$

- Suppose the random variables are bits*.

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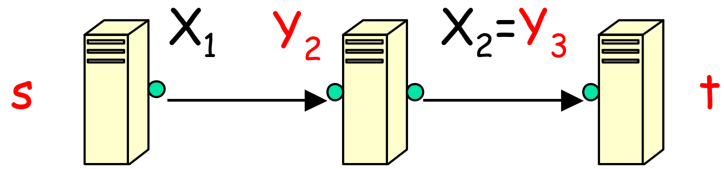
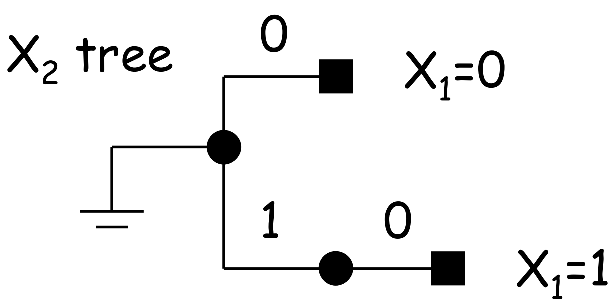
*Usually the X_t are packets, and not bits, but the following gives the general idea



- Guess: capacity is ½ bit/use (or packet/use) ?
- A "decompression" code at node 2:

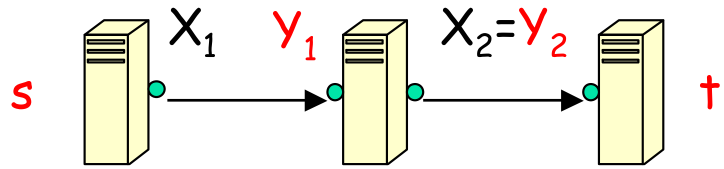


Node 2 transmits appropriate branch labels upon receiving X_1. For example:

X_1 = 0, 1, X0, 0, 1, X1, X1, X0

X_2 = 0, 0, 10, 0, 0, 10, 10, 10, 0

- 1st network edge: every X_2 word has one zero
- 2nd network edge: $R \leq 1/E[L_2] = 2/3$ bits/use !



- Better compression codes (e.g., Huffman codes, arithmetic source codes) achieve R = 0.773 bits/use with $\Pr[X_2=0] = 0.773$*.

- How can we understand this gain?
  Is 0.773 the capacity of this network?

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*This is when $\Pr[X_2=0] = h(\Pr[X_2=0])$, where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is Shannon's binary entropy function

Capacity Bounds

- The channel is memoryless and physically degraded so decode-and-forward (DF) achieves capacity (Cover & El Gamal, 1979):

$$C = \max_{P(x_1,x_2)} \min \left[I(X_1;Y_2|X_2),\ I(X_1X_2;Y_3) \right]$$

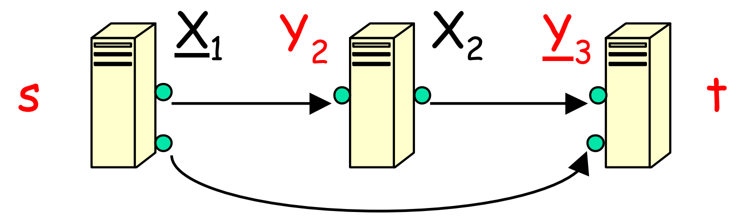
- Choose X_1 and X_2 independent and X_1 coin-tossing

$$I(X_1;Y_2|X_2) = H(Y_2|X_2) = \Pr[X_2=0]$$
$$I(X_1X_2;Y_3) = H(Y_3) = h(\Pr[X_2=0])$$

- Note: our new DF protocol based on source coding has <u>low-delay</u> and <u>variable-length codewords</u>
- Many relays, one path: capacity via DF (Aref, 1980)

3) Generalizations & Problems

- One relay, two paths:

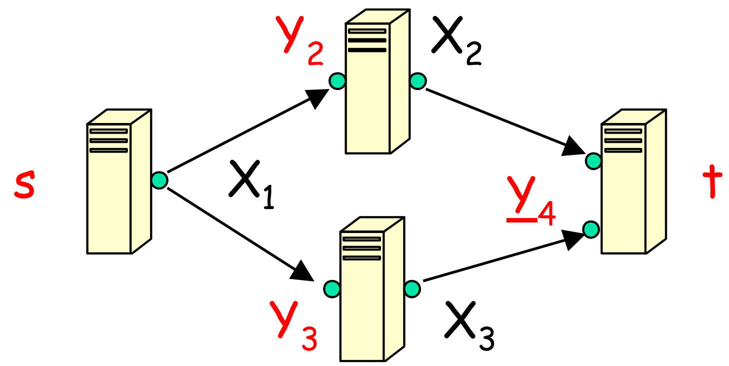


- A memoryless (non-degraded but) deterministic relay channel. The capacity is (El Gamal & Aref, 1982)

$$C = \max_{P(x_1,x_2)} \min \left[H(Y_2 Y_3 | X_2), H(Y_3) \right]$$

- Strategy: use "partial" DF which is a path-based protocol that superposes two DF schemes.
- Problem: what kind of source coding works here?

- Two relays, two paths (a Schein-Gallager network):



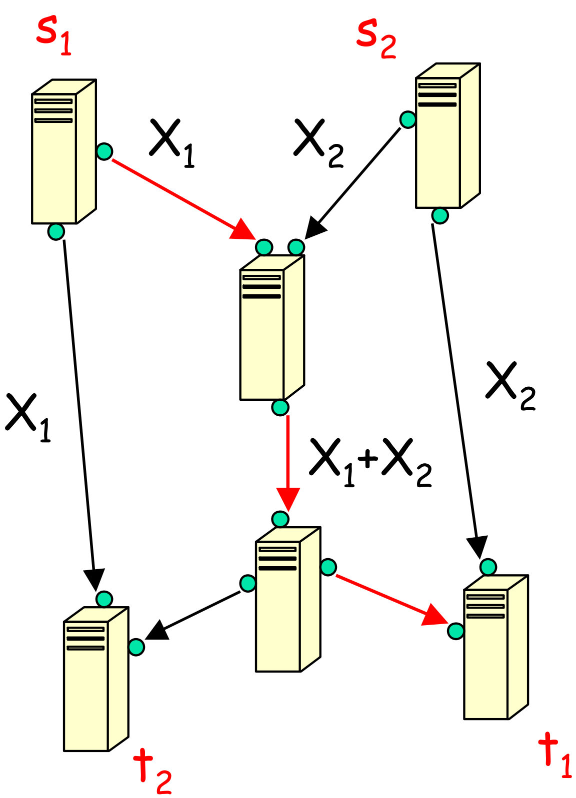
- Capacity is not known (even with no constraint at $\underline{Y}_4$)
- A path-based strategy (Gupta & Kumar, 2003) achieves $R=R_1+R_2$ where

$$R_1 = \min\left[I(U_2;Y_2|X_2),\ I(U_2X_2;\underline{Y}_4)\right]$$

$$R_2 = \min\left[I(U_3;Y_3|X_3),\ I(U_3X_3;Y_4|X_2U_2)\right]$$

$$P(u_2,u_3,x_2,x_3) = P(u_2,x_2)\ P(u_3,x_3)$$

- Many sources/paths, unicast/multicast:



- Suppose every node has a 1-port constraint
- Basic routing throughput:
  1/2 bit/use
- Basic network coding:
  2/3 bits/use
- Relay routing:
  0.732 bits/use*

*in general, one should combine network coding and relaying

*for packets, the gains are smaller but do permit “covert” communication

Summary

- Two Basic Open Problems:
  - Find the unicast and multicast capacities of deterministic relay networks.
    (Is the cut-set upper bound achievable or not?)
- Partial results:
  - No node constraints and no interference:
    - unicast: use routing (Ford & Fulkerson, 1956)
    - multicast: use network coding (Ahlswede, Cai, Li, Yeung, 2000).
  - No node constraints but with broadcasting:
    - unicast: use binning and routing (Aref, 1981)
    - multicast: use (modified) network coding (Ratnakar & K, 2005)