Power Control for Fading Multiple Access Channels with User Cooperation

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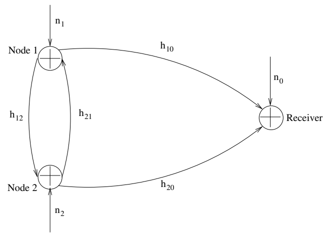
Introduction



$$Y_0 = h_{10}X_1 + h_{20}X_2 + n_0$$
$$Y_1 = h_{21}X_2 + n_1$$
$$Y_2 = h_{12}X_1 + n_2$$

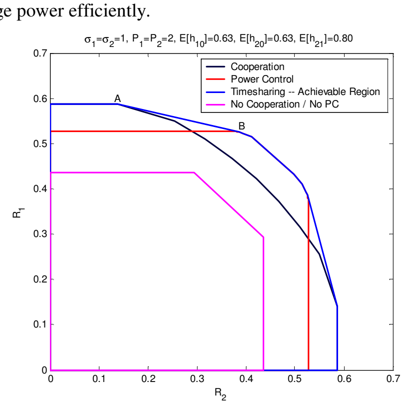
- Interference is information.
- Some versions of all transmitted signals are received by all nodes.
- User cooperation: exploit overheard information to jointly design encoding, transmit, routing policies.
- Building block towards the analysis of ad-hoc networks.

User Cooperation and Achievable Rates

- Block Markov superposition coding, followed by backward decoding
  - Build common information (X_{12}, X_{21})
  - Cooperatively send (U)
  - Inject new information (X_{10}, X_{20})

$$X_1 = p_{10}(\mathbf{h})X_{10} + p_{12}(\mathbf{h})X_{12} + p_{u1}(\mathbf{h})U$$
$$X_2 = p_{20}(\mathbf{h})X_{20} + p_{21}(\mathbf{h})X_{21} + p_{u2}(\mathbf{h})U$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available limited average power efficiently.



- User cooperation and power control separately achieve significant gains.
- We combine power control and user cooperation to find jointly optimal policies.

Joint Power Control and User Cooperation -- Optimization

- Achievable region of rates obtained by taking the union of achievable rates for every valid power control policy, which yields rather complicated expressions.
- The functions defining the boundary of this region are non-concave/convex in the vector of variables $\mathbf{p}(\mathbf{h})=[p_{10}(\mathbf{h}), p_{12}(\mathbf{h}), p_{U1}(\mathbf{h}), p_{20}(\mathbf{h}), p_{21}(\mathbf{h}), p_{U2}(\mathbf{h})]$.
- We first simplify the optimization problem using our main result below:

- **Theorem 1** Let the effective channel gains normalized by the noise powers be defined as $s_{ij} = h_{ij}/\sigma^2_j$. Then, for the power control policy $\mathbf{p}(\mathbf{h})$ that maximizes the sum rate, we need
- $p_{10}(\mathbf{h}) = p_{20}(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} > s_{20}$
- $p_{10}(\mathbf{h}) = p_{21}(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} \le s_{20}$
- $p_{12}(\mathbf{h}) = p_{20}(\mathbf{h}) = 0$, if $s_{12} \le s_{10}$ and $s_{21} > s_{20}$
- $p_{12}(\mathbf{h}) = p_{21}(\mathbf{h}) = 0$ OR $p_{10}(\mathbf{h}) = p_{21}(\mathbf{h}) = 0$ OR $p_{12}(\mathbf{h}) = p_{20}(\mathbf{h}) = 0$, if $s_{12} \le s_{10}$ and $s_{21} \le s_{20}$

Implications of Zero-Power Components

- Block Markov superposition coding is simpler than originally thought.
  - Each transmitter either sends a cooperation signal or fresh information, but not both
- The choice at each channel state "only" depends on the channel state.
  - Channel statistics, power constraints play no role in deciding which signals to Xmit.
  - Except for the last case, which usually has very insignificant probability.
- The achievable rate expressions are greatly simplified, and are now concave.
- This simplified coding policy not only maximizes the sum rate, but also the individual rate constrains on $R1$ and $R2$, and is optimal in terms of the entire rate region.
- Concave optimization problem over a convex constraint set, but non-differentiable.

Simplified Optimization Problem

- Points on the rate region boundary can be obtained by maximizing $C_\mu = \mu_1 R_1 + \mu_2 R_2$.

$$\max_{\mathbf{p}(\mathbf{h})} \ \mu_1 R_1 + \mu_2 R_2 \quad \text{s.t.} \ E[p_1(\mathbf{h})] \le P_1, \ E[p_2(\mathbf{h})] \le P_1$$

- $(R_1;R_2)$ is the corner of the pentagon obtained for a given power allocation policy.
  - Example: for $s_{10}<s_{12}$, $s_{20}<s_{21}$,

$$R_1 < E[\log(1+s_{12}p_{12}(\mathbf{h}))]$$
$$R_2 < E[\log(1+s_{21}p_{21}(\mathbf{h}))]$$
$$R_1 + R_2 < \min\left\{ E\left[\log(1+s_{10}p_1(\mathbf{h})+s_{20}p_2(\mathbf{h})+2\sqrt{s_{10}s_{20}p_{U1}(\mathbf{h})p_{U2}(\mathbf{h})}\right], E\left[\log(1+s_{12}p_{12}(\mathbf{h}))+\log(1+s_{21}p_{21}(\mathbf{h}))\right]\right\}$$

•Sum rate not differentiable where the arguments of the min are equal.

Optimum Power Allocation via Subgradient Method

- Gradient of the objective function does not exist everywhere, find subgradient $\mathbf{g}$ instead

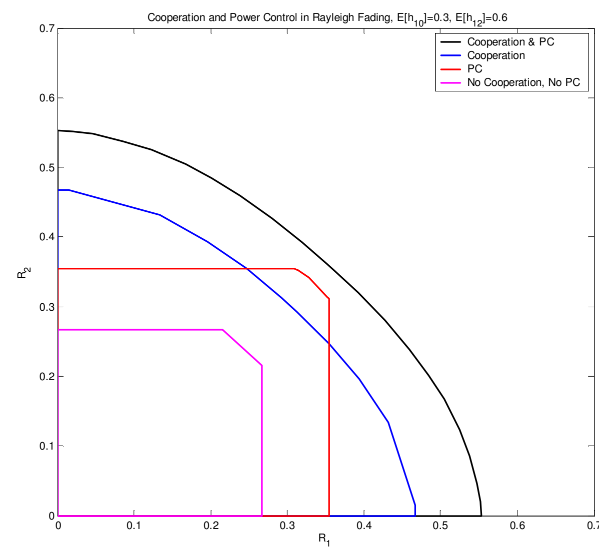
$$C_\mu(\mathbf{p}') \le C_\mu(\mathbf{p}) + (\mathbf{p}'-\mathbf{p})^T\mathbf{g}$$

- Use projected subgradient method to maximize C_μ

$$\mathbf{p}(k+1) = [\mathbf{p}(k)+\alpha_k\mathbf{g}_k]^+$$

- Does not necessarily give monotonically increasing function values.
- Yet, provably converges for a diminishing stepsize α_k [Shor].

Resulting Achievable Rate Region



- Significantly enlarged rate region AND simpler user cooperation scheme!

Summary and Conclusions

- Characterized the power control policies that are jointly optimal with Block Markov superposition coding.
- Using sub-gradient methods, obtained optimal power levels and corresponding rate region.
- Encoding and decoding is significantly simplified.
  - Transmitters send either cooperation or fresh information signals, but not both.
- Joint usage of cooperative diversity and time diversity:
  - Major improvements in capacity.
- Optimal power policies also dictate MAC and routing policies
  - Cross layer design.

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