

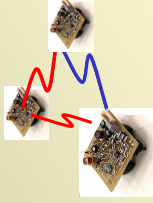
# Cooperative Strategies for the Relay Channel

In the Low SNR Regime  
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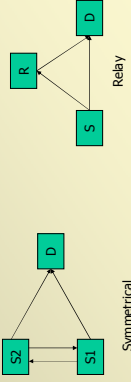
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## Cooperative Communication

- Transmit diversity: may not be practical
  - most handsets (size)
  - wireless sensor network (size, power)
- Single-antenna mobiles share their antennas
  - Virtual Multiple-Antenna system



- Two scenarios:



Symmetrical

1

## Upper Bound on the Performance

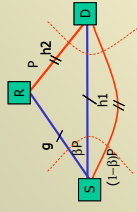
- Min-Cut Max-Flow Bound:
 
$$C_{\text{relay}}(g, h_1, h_2) \leq \max_{\beta} (\|h_1\|^2 P + \min(\|g\|^2 \beta P, \|h_2\|^2 P))$$

$$= (\|h_1\|^2 P + \min(\|g\|^2 P, \|h_2\|^2 P))$$
- The corresponding bound on the outage capacity
 
$$C_{\epsilon, \text{relay}} \leq \sqrt{\epsilon} SNR$$



4

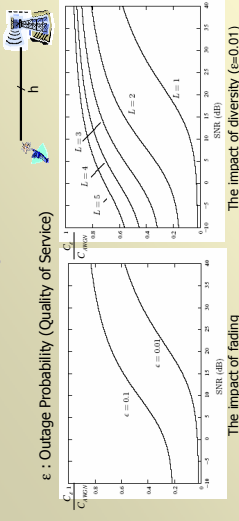
## Full CSI Scenario



- Allocate some power for beam forming
 
$$C_{\text{relay}}(g, h_1, h_2) \leq \max_{\beta, \rho} (\|h_1\|^2 \beta P + \min(\|h_1\|^2 (1-\beta)(1-\rho^2)P + \|g\|^2 \beta \rho, \|h_2\|^2 P + \|h_1\|^2 (1-\beta)P + 2\|h_1\| \|h_2\| \rho \sqrt{(1-\beta)P}))$$
- For any choice of  $\beta, \rho$  it can be achieved by BAF + Beam forming

7

## Diversity at Low SNR



- The impact of diversity is more significant
- Outage Capacity:  $\epsilon SNR = \epsilon C_{\text{AWGN}}$

2

## Cooperative Schemes

1. Decode-Forward:
 
$$R_{\epsilon, \text{DF}} \approx \sqrt{\frac{2}{3}} \epsilon SNR$$
  - Exploits Diversity

MFMC Upper Bound	$C_{\epsilon, \text{relay}} \leq \sqrt{\epsilon} SNR$
Decode-Forward	$R_{\epsilon, \text{DF}} \approx \sqrt{\frac{2}{3}} \epsilon SNR$

-Not Optimal, Why?  
Cut-set:  $C_{\text{relay}}(g, h_1, h_2) \leq (\|h_1\|^2 P + \min(\|g\|^2 P, \|h_2\|^2 P))$

2. Amplify-Forward:
 
$$R_{\epsilon, \text{DF}} \approx \epsilon SNR$$
  - No Diversity

-Relay becomes useless by injecting more noise than signal.

5

## The Gain From Full CSI

- With Full CSI:
 
$$C_{\epsilon, \text{relay}} \approx 1.04 \times \sqrt{\epsilon} SNR$$
 the gain is small.
- The reason is that in previous optimization only small amount of power will be allocated for beam forming (6%)
- In the MISO case, the gain is 3dB as all the power is used for beam forming

8

## The Relay Scenario



- Rayleigh fading channels + AWGN noise (variance 1)
- Slow fading: Outage event
- Half-Duplex constraint
- Channel State Information available at the receiver only
- Average power constraint (P) on both S and R

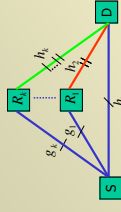
3

## A Better Protocol?

- Bursty Amplify-Forward:
  - To have less noisy observation at the Relay, source transmits rarely (a fraction of the time) but with high power:
 
$$SNR_{\text{eff}} = \frac{SNR}{\alpha}$$
 be large
  - In order to be power efficient, we should pick a such that the effective rate is small:
 
$$R_{\text{eff}} = \frac{R}{\alpha}$$
 be small
  - Can we pick such a ?
    - Yes, because at low outage probability  $\frac{R}{SNR}$  is small.
  - The achievable rate of this scheme:  $R_{\epsilon, \text{BAF}} \approx \sqrt{\epsilon} SNR$
  - Why optimal?
 
$$C_{\text{relay}}(g, h_1, h_2) \leq (\|h_1\|^2 P + \min(\|g\|^2 P, \|h_2\|^2 P)) / N$$

6

## Network with k Relays



- Min-Cut Max-Flow Bound:
 
$$C_{k\text{-relays}}(\vec{h}, \vec{g}) \leq \|h_1\|^2 P + \sum_{i=1}^k \min(\|g_i\|^2, \|h_{i+1}\|^2) P$$
- The corresponding bound on the outage capacity
 
$$C_{\epsilon, k\text{-relays}} \leq k+1 \sqrt{\frac{(k+1)!}{2^k}} \epsilon SNR$$
- The cut-set is achieved by BAF protocol on each relay

9