UNIVERSITY OF MINNESOTA TWIN CITIES
Dept. of Electrical & Computer Engineering

Recovering Diversity in Regenerative Cooperative Systems

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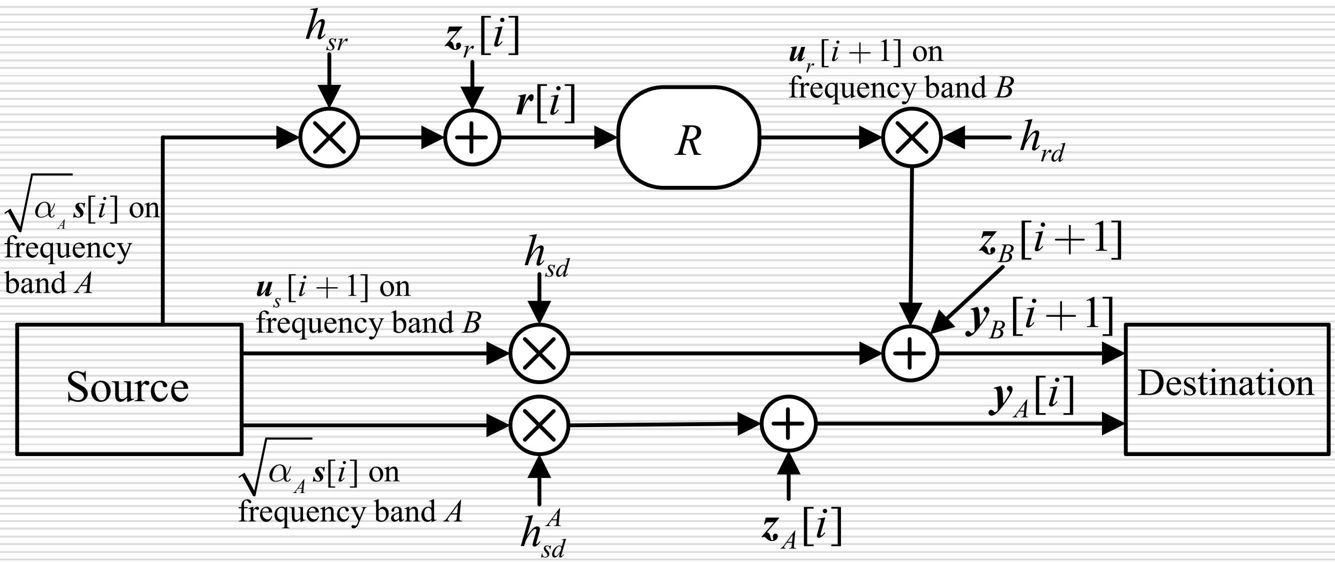
Paul Anghel, Mostafa Kaveh
Department of Electrical and Computer Engineering,
University of Minnesota

Geert Leus
Faculty of Electrical Engineering,
Mathematics and Computer Science
Delft University of Technology

The DF System Model



Assumptions: relay R knows h_{sr} and the destination knows h_{rd}

Source and relay cooperate using an Alamouti space-time code:

$$\begin{cases} \mathbf{y}_A = h_{sd}^A \sqrt{\alpha_A} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \mathbf{z}_A \\ \mathbf{y}_B = h_{sd} \sqrt{\alpha_B} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + h_{rd} \sqrt{\alpha_R} \begin{bmatrix} \hat{s}_1^* \\ -s_0^* \end{bmatrix} + \mathbf{z}_B \end{cases}$$

Performance of the Alamouti Receiver

- After processing:

$$x = (\alpha_R |h_{rd}|^2 \theta_1 + \alpha_B |h_{sd}|^2) s_0 + \underbrace{\sqrt{\alpha_R \alpha_B} h_{rd} h_{sd}^* (\theta_1 - 1) s_1}_{\text{INTERFERENCE}} + \underbrace{\sqrt{\alpha_B} h_{sd}^* z_0 - \sqrt{\alpha_R} h_{rd} z_1}_{\text{NOISE}}$$

- With unknown relay error probability at the destination:

$$s_0 = 1 \text{ only if } x + h_{sd}^{A*} \sqrt{\alpha_A} y_A \geq 0$$

- We show that the "out of the box Alamouti" loses diversity:

$$P_1(\varepsilon_R) = \Pr\left(\mathcal{R}\left\{x + h_{sd}^{A*} \sqrt{\alpha_A} y_A\right\} < 0 \middle| s_0 = 1\right) = O\left(\left(\frac{\varepsilon}{N_0}\right)^{-G_{sr}}\right)$$

Yanikomeroglu et. al. – also observes the loss of diversity

Barbarosa et. al. – comments on the loss of diversity in a space-time coding setup

Proposed DSTC Receiver with One Relay

Problem

Design the amplification at the relay for full diversity transmissions

- If the relay's error probability is known at the destination

$$\max_{\alpha_R} \gamma_t(\alpha_R) = \frac{\left(\left(1-2P_R\right)\alpha_R\gamma_{rd} + \alpha_B\gamma_{sd}\right)^2}{4P_R\left(1-P_R\right)\alpha_R^2\gamma_{rd}^2 + 4P_R\left(1-P_R\right)\alpha_R\gamma_{rd}\alpha_B\gamma_{sd} + \alpha_R\gamma_{rd} + \alpha_B\gamma_{sd}} + \alpha_A\gamma_{sd}^A$$

subject to $\alpha_R \le \varepsilon_R$

- Solution: $\alpha_R^0 = \arg\max_{\alpha_R \in \{0,\varepsilon_R\}} \{\gamma_t(\alpha_R)\}$

Remark 1: the optimization result is used to reduce the set of amplifications at the relay from $[0,\varepsilon_R]$ to the set $\{0,\varepsilon_R\}$.

Remark 2: the system does not switch between α_R=0 and $\alpha_R = \varepsilon_R$ based on maximum SNR.

Minimum Probability of Error Approach

- Proposed amplification: $\alpha_R^0 = \arg \min_{\alpha_R \in \{0,\varepsilon_R\}} \{P_1(\alpha_R)\}$

  - requires a feedback channel!

  Not the optimum approach but $P_1(\alpha_R) = O\left(\left(\frac{\varepsilon_R}{N_0}\right)^{-\left(G_{sd}+G_{sd}^A+\min(G_{sr},G_{rd})\right)}\right)$

- A sensible performer: $\alpha_R^1 = \frac{(1-2P_R)\varepsilon_R}{1+4P_R(1-P_R)(\varepsilon_R+\alpha_B)E[\gamma_{rd}]}$

  - less subjective to errors on the feedback channel!

Extension to a Two Relay Setup

- Known relay error probabilities at the destination

$$\max_{\alpha_{R_1},\alpha_{R_2}} \gamma_{t2}\left(\alpha_{R_1},\alpha_{R_2}\right)$$

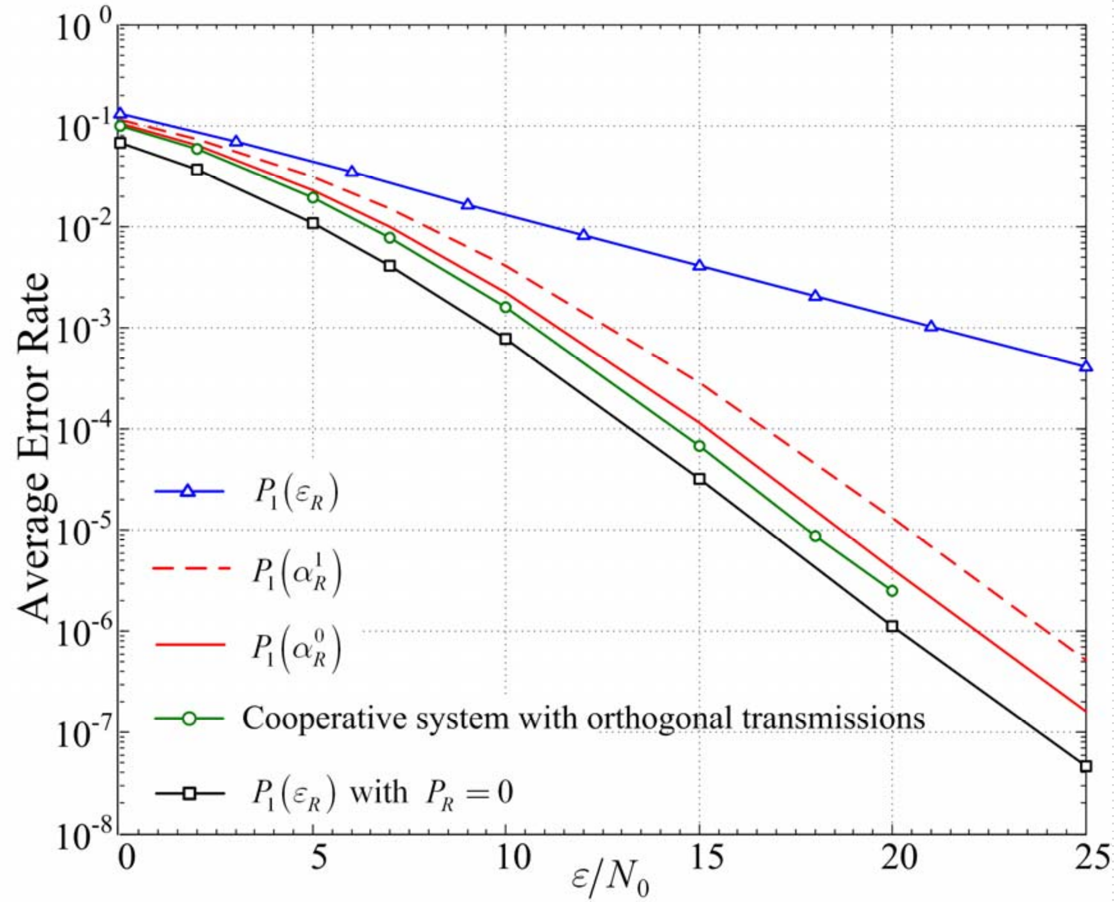
subject to $\alpha_{R_1} \le \varepsilon_{R_1}$ and $\alpha_{R_2} \le \varepsilon_{R_2}$

- Solution: $\gamma_{t2}^{0} = \max\left\{\gamma_{t2}(\varepsilon_{R_1},0),\gamma_{t2}(0,\varepsilon_{R_1}),\gamma_{t2}(\varepsilon_{R_1},\varepsilon_{R_2})\right\}$
- Proposed amplifications:

$$\left[\alpha_{R_1}^{0},\alpha_{R_2}^{0}\right] = \arg\min\left\{\left[\alpha_{R_1},\alpha_{R_2}\right] \in \left\{\left[\varepsilon_{R_1},0\right],\left[0,\varepsilon_{R_2}\right],\left[\varepsilon_{R_1},\varepsilon_{R_2}\right]\right\} \,\middle|\, P_2\left(\alpha_{R_1},\alpha_{R_2}\right)\right\}$$

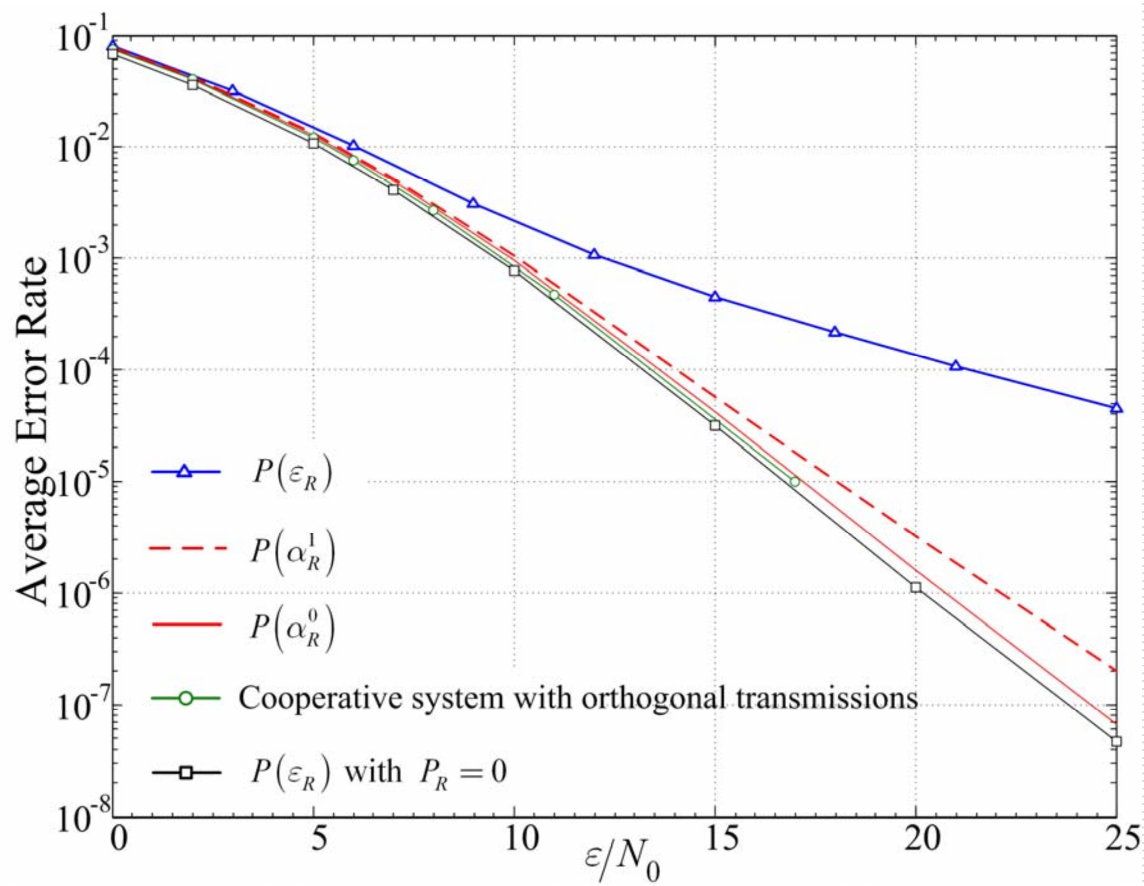
- Similar w/ one relay case: $\alpha_{R_q}^{1} = \dfrac{\left(1-2P_{R_q}\right)\varepsilon_{R_q}}{1+4P_{R_q}\left(1-P_{R_q}\right)\left(\varepsilon_{R_1}+\varepsilon_{R_1}\right)E\left[\gamma_{r_q d}\right]}, \quad q=1,2$

Error Rate of the DSTC System



Equally balanced channels

Error Rate of the DSTC System (con't)



Unequally balanced channels: $E[\gamma_{sr}] = 10E[\gamma_{rd}]$

Feedback Rate